# Statistical Analysis of Connectivity in Unidirectional Ad Hoc Networks

Venugopalan Ramasubramanian*
Department of Computer Science
Cornell University
Ithaca, NY 14850
ramasv@cs.cornell.edu

Daniel Mossé†
Department of Computer Science
University of Pittsburgh
Pittsburgh, PA
mosse@cs.pitt.edu

## Abstract

*A* unidirectional link *exists in an ad-hoc network when a node B is within the transmission range of another node A while node A cannot directly hear node B. However, a* reverse route *from B to A might exist, by going through multiple nodes. Unidirectional links may exist in an ad hoc network due to variation in transmission power of different nodes, noise or other signal propagation phenomena, and heterogeneity in transmission hardware of nodes in the network. In this paper, we statistically analyze the connectivity of ad hoc networks in the presence of unidirectional links. We generate several random topologies employing two models and study the connectivity of the sub-graphs formed by including unidirectional links of different reverse-route lengths. We observe from this analysis that the connectivity has a heavy-tail distribution and that using only bidirectional links could cause partitions in the network. This analysis also shows that the inclusion of unidirectional links with short reverse-routes (2-3 hops) is often sufficient to restore good connectivity in unidirectional networks.*

**Keywords:** *unidirectional, routing, connectivity, ad hoc network.*

## 1 Introduction

A network of mobile nodes using peer-to-peer communication and without a fixed communication infrastructure is called an *ad hoc network*. The lack of infrastructure allows such networks to be deployed quickly. Hence, they are very useful in disaster recovery, collaborative work, rescue operations and military surveillance. However, the nodes in an ad hoc network are typically limited by power, memory, bandwidth and computation capabilities.

An enabling technology is the ability of wireless network cards to transmit at different power levels. Allowing these devices to transmit at lower power levels would help to prolong their lifetime. In most foreseeable environments, hand-held devices, laptops running on battery power, laptop hooked to a power supply, base station transmitters are all interacting devices with inherently different power supplies. It is advantageous for these devices to operate at their own optimal power for communication. Further, it is advantageous to allow these devices to lower their transmission power in a very dense environment to decrease congestion and to increase their transmission power in a sparse environment to increase connectivity.

A fundamental problem of allowing nodes to transmit with different transmission powers is the creation of unidirectional links in the network. For example, if node $A$ is transmitting at higher power than other nodes and node $B$ is within the transmission range of node $A$ while node $A$ cannot hear node $B$, the link $A \rightarrow B$ is unidirectional. Even when nodes are transmitting at the same power, unidirectional links may be created due to increased collisions or noise affecting packet reception at one node more than another. Unidirectional links created by these causes are often local and transient.

The presence of unidirectional links severely affects the functionality of an ad hoc network at various layers. In the media-access layer, congestion avoidance schemes such as RTS-CTS and other services such as link-status sensing are severely hampered. Hop-level acknowledgments at the link layer cannot be directly sent to the upstream node of a unidirectional link. In network layer routing protocols, the efficient operation of route discovery is severely impaired. Additional route discoveries may have to be initiated to forward route replies. Even protocols such as AODV [5] that choose to ignore unidirectional links are no longer efficient unless they are able to accurately identify the unidirectional links. Other protocols such as DSR [3] have to use expensive mechanisms (multi-hop acknowledgments) to maintain routes with unidirectional links.

However, the most important impact of unidirectional links is in the connectivity of the ad hoc network. In this paper, we examine the impact of unidirectional links on this fundamental property. We perform a statistical analysis by randomly generating several topologies with widely distributed parameters of

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transmission power, node density and number of nodes. We introduce unidirectional links based on two models: the P-model that introduces unidirectional links randomly according to a probability distribution function and the D-model that introduces unidirectional links by varying the transmission range of the nodes. Our analysis of the connectivity of these topologies provides interesting observations that can be used to guide the design of routing strategies to handle unidirectional networks. To the deployers of ad hoc networks, it provides directions to minimize the impact of unidirectional links on existing strategies.

In Section 2 we describe the efforts by others to study the impact of unidirectional links to routing protocols and in Section 3 we define the terms used in this paper. The extensive statistical analysis of topologies with unidirectional links is presented in Section 4. We conclude the paper in Section 5.

## 2 Related Work

Several techniques have been explored to support handling of unidirectional links in routing protocols. Ad hoc On demand Distance Vector routing protocol, AODV [7], has been extended to work in an asymmetric network by identifying and ignoring the unidirectional links. Asymmetric links are detected as they are encountered and registered in a *black list* [5] in order to be ignored in the future. A shortcoming of the blacklist approach is that it excludes unidirectional links, perhaps causing network disconnection. Dynamic Source Routing protocol, DSR [3], is designed to operate in the presence of unidirectional links. Separate discovery and maintenance is performed for both the forward and the reverse route between the source and the destination. Multi-hop link level acknowledgements are sent by maintaining reverse route for every link in an active route. There has been little work on quantifying the costs of unidirectional links in ad hoc networks.

Discovery and maintenance of reverse routes is a popular technique to support unidirectional links in routing protocols. A proactive link-state routing protocol that employs reverse routes is described in [1]. This protocol builds and maintains an *inclusive cycle* for each unidirectional link. The inclusive cycle is formed by the unidirectional link and its reverse route. The link-state updates are then sent along these reverse routes. The Sub Routing Layer, SRL [8], provides a bidirectional abstraction of an asymmetric network to the routing protocols. SRL employs a locally proactive distance vector algorithm to discover and maintain reverse routes efficiently for unidirectional links. Routing protocols can use many services provided by SRL such as neighbor discovery, multi-hop acknowledgements and reverse route forwarding to perform routing in an asymmetric network. The efficiency of these schemes depends on the length of the reverse routes. SRL is very efficient for maintaining short reverse routes (2-3 hops). One of the key observations made in this paper is that short reverse routes are often sufficient to obtain good connectivity in asymmetric networks.

Even though several techniques to support routing in asymmetric networks have been explored, an in-depth analysis of the impact of unidirectional links to the connectivity of the network has not been adequately performed to the best of our knowledge. In [6], the authors qualitatively examine several problems associated with distance vector routing in the presence of unidirectional links. In particular, they show that the size of routing messages exchanged would increase from $O(n)$ to $O(n^2)$, where $n$ is the size of the network. In this paper, we restrict ourselves to a quantitative analysis of connectivity in the presence of unidirectional links and do not examine other impacts on routing protocols.

In [4] an analysis similar to ours is presented. However, the authors only discuss the connectivity of the unidirectional networks in comparison with the sub-graph that is connected completely with bidirectional links. In this paper, we present an analysis of the connectivity of different categories of subgraphs formed by the inclusion of unidirectional links with different reverse route lengths. We analyze a wider class of scenarios and at a much greater depth, which enables us to make important observations that improve the efficiency of unidirectional routing protocols.

In [2] the authors present an empirical study of broadcast protocols in a large scale multi-hop wireless networks. Experiments performed by deploying 185 sensor nodes in a uniform grid indicate a high incidence of asymmetry even when all the nodes transmit at the same power. In particular, the authors in [2] report that 5% to 15% of links were unidirectional during their experiments. Clearly, in the presence of heterogeneity in transmission power this scenario could be further aggravated.

These observations suggest that presence of unidirectional links is a significant problem in real life and efficient mechanisms are needed to handle them. In this paper, we explore how asymmetry affects the connectivity of a network.

## 3 Notations and Definitions

The topology of a network is considered to be a directed graph, $D = (V, E)$, where $V$ is the set of nodes in the network and $E$ the set of links in the network. A *link* $A \to B$ exists between two nodes $A$ and $B$ if $B$ is within the transmission range of $A$. A link $A \to B \in E$ is said to be *bidirectional* if $B \to A \in E$ and *unidirectional* if $B \to A \notin E$. The *reverse route* of a link $A \to B$ is defined as the shortest directed path from $B$ to $A$ and the length of this shortest path is the *reverse route length* of the link. If no such path exists between $B$ and $A$ then the reverse route and the reverse route length are not defined. Thus, a bidirectional link would have a reverse route length of 1 hop. If the network is strongly connected, then every link would have a reverse-route.

We categorize the links in the network based on the reverse route length parameter. A link with reverse route length $r$ is said to be an *r-link*. Thus *1-links* represent the set of all bidirectional links in the network. The *r-graph* of a network is defined as the sub-graph consisting only of the links with reverse route of length at most $r$. By this definition, the $\infty$-*graph* of the network would consist of all the links that have a reverse route. Hence, each component in the $\infty$-graph would be strongly connected. The *1-graph* of a network would only include the bidirectional links and hence represent the routing structure used by routing protocols [5] that route only using the bidirectional links. We also use the term *bi-graph* as an alternate to 1-graph in this paper.

# 4 Topology Analysis of Unidirectional Networks

In this section, we study the impact of the presence of unidirectional links on network characteristics. Since it is difficult to perform rigid mathematical analysis and obtain useful closed form expressions, we resort to performing simulations in order to statistically analyze network topologies.

We generate random topologies with unidirectional links and analyze them for relevant network properties including connectivity in the presence and absence of unidirectional links. Each of these topologies represents an instantaneous snapshot of a dynamically varying network. This elaborate statistical analysis of several topologies offers several valuable insights for efficient routing in unidirectional network.

Several topologies are randomly generated with parameters commonly used in simulations of routing protocols (based on default parameters of Glomosim [9]). Each topology consists of 100 nodes placed in a square field randomly with a uniform probability distribution. The area of the square field is varied by changing the density of the nodes. The density of nodes is varied starting from 30 nodes/sq. km. to 100 nodes/$km^2$ in steps of 10. We also performed similar trials with rectangular fields and the results observed were qualitatively similar to what is presented here.

The links between the nodes are established based on two models. In the first model, each node is assigned a range of 220m (this corresponds to the nominal transmission range of WaveLan radios), generating a bidirectional topology. Each bidirectional link is then converted into a unidirectional link with a probability P, which is varied on a linear scale from 0 to 0.4 in steps of 0.05. This so called *P-model* tries to mimic topologies with unidirectional links due to noise, collisions and other factors.

The second model, the *D-model*, generates a more realistic topology by assigning different transmission ranges to each node. We do this by defining a metric called diversity. The *diversity*, D, of a topology is defined as the difference between the maximum and minimum transmission ranges of the nodes in the network. According to this model each node was assigned a transmission range picked randomly (uniform distribution) from the interval $[N - \frac{D}{2}, N + \frac{D}{2}]$, where N is the nominal range set to 220m. Thus a diversity of 0 produces a topology with only bidirectional links. We vary the value of the diversity between 0m and 320m in steps of 40m. In order to simulate radios with continuous and discrete steps of transmission ranges, we varied the granularity in picking the transmission ranges from 1m to 60m. However, this produced no noticeable difference in the results.

For each set of parameters described above we randomly generate 500 topologies and analyze them statistically. The following sub-sections describe the observations made from this analysis. In the graphs that are described below, each data point corresponds to one experiment with 500 trials of random topologies. The error bars plotted in the graph show the 99% confidence interval of the values. We repeated these experiment with 50, 200, and 400 nodes in the topology for the same values of density but found that the average connectivity is the same as that for 100 nodes, at each density.

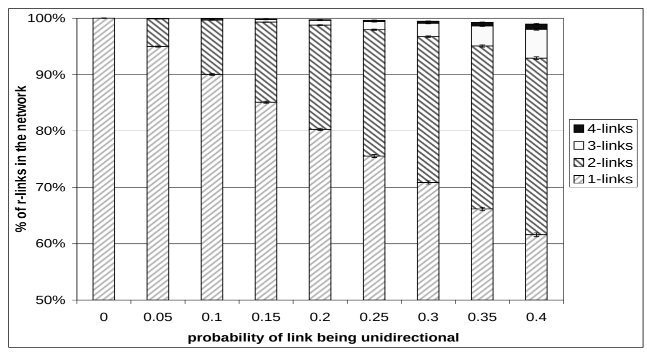


**Figure 1. Average distribution of r-links in the network. P-model, density = 50 nodes/$km^2$**

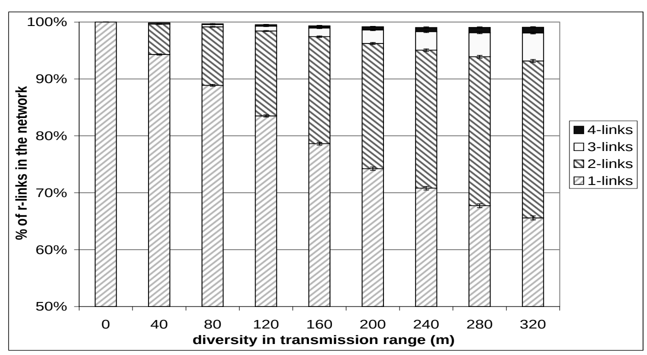


**Figure 2. Average distribution of r-links in the network. D-model, density = 50 nodes/$km^2$**

### Link Distribution Statistics

The first property of topology we discuss is the aggregate link characteristics for the different topologies.

The graphs shown in Figures 1 and 2 show the percentage of r-links in the network for different values of $r$ in the P-model and D-model respectively for topologies with density

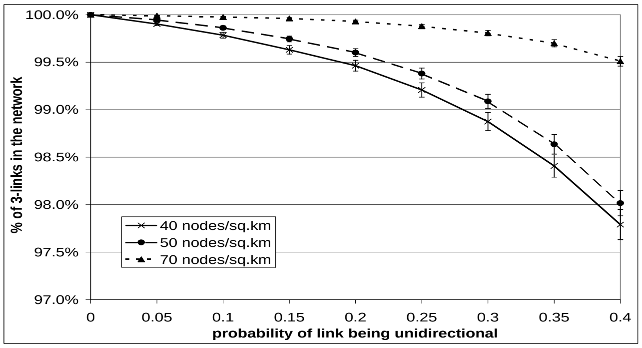


**Figure 3. Average distribution of 3-links in the network. P-model.**

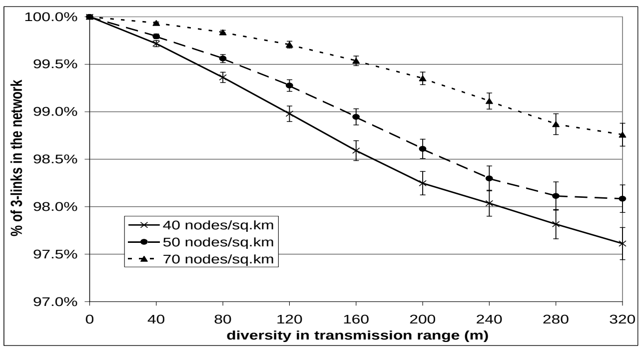


**Figure 4. Average distribution of 3-links in the network. D-model.**

of 50 nodes/$km^2$. The parameters, diversity D and probability P, of the respective models are plotted on the x-axis. The y-axis shows the cumulative contribution of the links (in percentage) with each value of reverse route length to the total number of links with reverse routes in the topology. The contribution of r-links for $r > 4$ is negligible and hence not shown in the graphs.

Let us first observe the percentage of bidirectional links (1-links) in the topology. The graph in Figure 1 shows a linear decrease in the percentage of 1-links from 100% to 62% as P varies from 0 to 0.4. This is expected because in the P-model we converted bidirectional links to unidirectional with probability P. The value at P equal to 0.4 is slightly more than 60% because the links with no reverse route are excluded. The graph in Figure 2 shows the contribution of bidirectional links in the D-model. We notice a decrease from 100% to 66% as D increases from 0m to 320m although it is not linear.

The important observation to be made from these graphs however is the contribution of links with small $r$ ($r \leq 3$) to the network. Both the models indicate that the contribution of r-links decreases sharply with increasing $r$. The graphs in Figures 3 and 4 plot the average contribution of 3-links (i.e., reverse route length less than 4) for three different densities. These graphs indicate that for both models, up to 97% of the links have very short (i.e., $\leq 3$) reverse route length for density above 40 nodes/$km^2$.

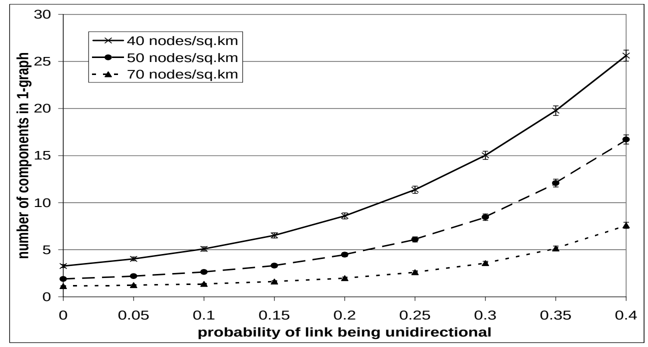


**Figure 5. Average number of connected components in the bi-graph. P-model.**

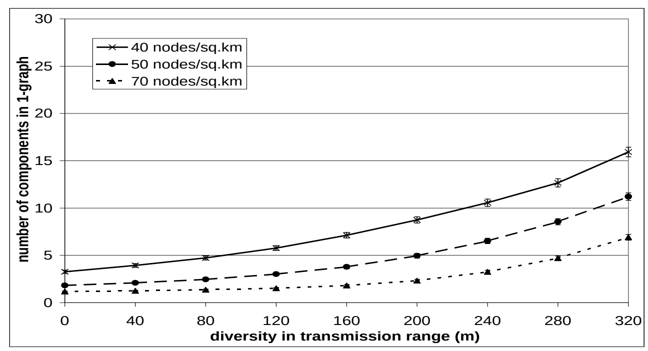


**Figure 6. Average number of connected components in the bi-graph. D-model.**

## Connectivity Statistics

We next examine the most important characteristic of the network, namely connectivity. The graphs in Figures 5 and 6 plot the average number of connected components in the bi-graph of the topologies against parameters P and D of the two models respectively. It can be observed from Figure 6 that the average number of connected components increases as the diversity increases, from about 2 to 11 for a density of 50 nodes/$km^2$. Similarly, the average number of connected components increases with increasing values of probability in Figure 5. Clearly, this increase in the number of connected components is lower at higher density. Nevertheless, a very high density is required to ensure good bidirectional connectivity for all the scenarios.

The graphs in Figures 7 and 8 show the average size of the largest connected component of the bi-graph for the two models at different densities. These graphs indicate a general decrease in the largest component at high diversity. For example, at a density of 50 nodes/sq.km, the average size of the largest component drops from about 97 to 82. This amounts to a decrease of about 15% in the size of the largest component. The

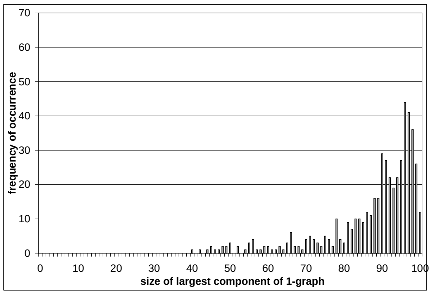


Figure 9(a): Histogram of the size of the largest component in the bi-graph. Diversity = 200m, density = 50 nodes/$km^2$

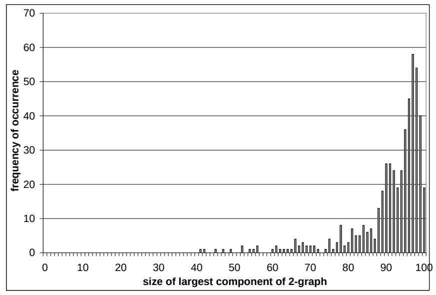


Figure 9(b): Histogram of the size of the largest component in the 2-graph. Diversity = 200m, density = 50 nodes/$km^2$

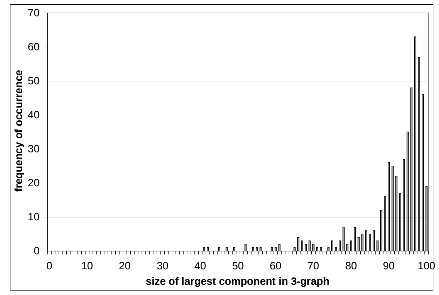


Figure 9(c): Histogram of the size of the largest component in the 3-graph. Diversity = 200m, density = 50 nodes/$km^2$

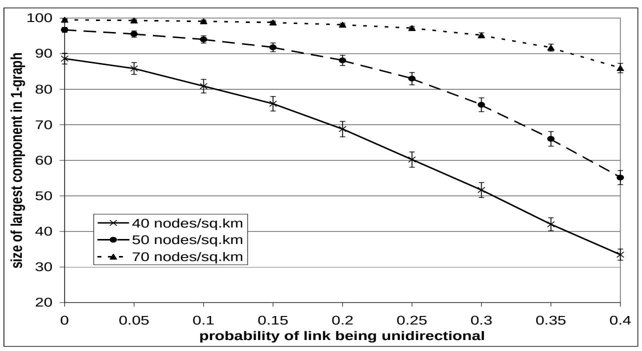


**Figure 7. Average size of the largest connected component in the bi-graph. P-model.**

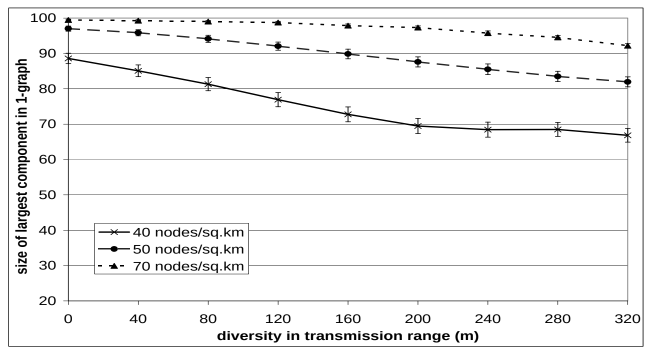


**Figure 8. Average size of the largest connected component in the bi-graph. D-model.**

decrease is significantly greater in the P-model compared to the D-model. The reason for this is that as bidirectional links are converted to unidirectional links in the P-model, the total number of links in the network decreases. Consequently, at high probabilities the network gets more and more disconnected.

We next take a detailed view of the statistics by looking at the distribution of these values. The graphs in Figures 9(a) to 9(c) plot the frequency distribution (histogram) of the size of the largest component in the r-graph ($r = 1, 2, 3$) of the 500 randomly generated topologies with diversity 200m and density 50 nodes/$km^2$. The x-axis gives the size of the largest component (recall that each topology has 100 nodes) while the y-axis shows the frequency, that is, the number of topologies with the corresponding size for the largest component.

The graph in Figure 9(a) indicates heavy clustering of values towards the right implying that the size of the largest component is very high (90s) almost always. However, the discerning feature of the graph is the heavy tail of the distribution growing to very low sizes for the largest component. This suggests that occasionally the unidirectional links play a very vital role in the connectivity of the network. The frequency distribution for the P-model (not included due to similarity of results) shows a heavy tail distribution as well.

The graphs in Figures 9(b) and 9(c) display the histogram of the size of largest component in the 2-graph and 3-graph of the topologies analyzed. The values can be seen to shift to the right as $r$ increases, showing that the connectivity improves significantly when 2-links are included. Inclusion of 3-links also increases the connectivity of the network although not as much. More importantly, the tail in the distribution is lighter indicating that the standard deviation of the size of the largest component gets smaller as $r$ increases.

The overall effect can be clearly seen in the graphs in Figure 10(a) and Figure 10(b), which show the average values of the size of the largest connected component for different values of $r$ in the P-model and D-model, respectively. The values shown are normalized to the average size of the largest component in the $\infty$-graph. The improvement in connectivity in the 2-graph compared to the bi-graph can be observed to be quite significant. The average size of the largest component of the 3-graph is at least 97% in the D-model and 95% in the P-model. The improvement in the connectivity in the $r$-graphs for $r > 3$ becomes insignificant.

The graphs in Figure 10(c) and Figure 10(d) plot the trends of the normalized average size of the largest component of 3-graph in the P-model and D-model. At densities greater than

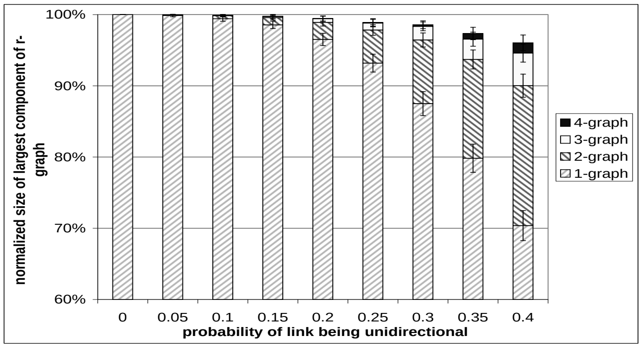


Figure 10(a): Average size of the largest connected component in r-graph. P-model, density = 50 nodes/$km^2$

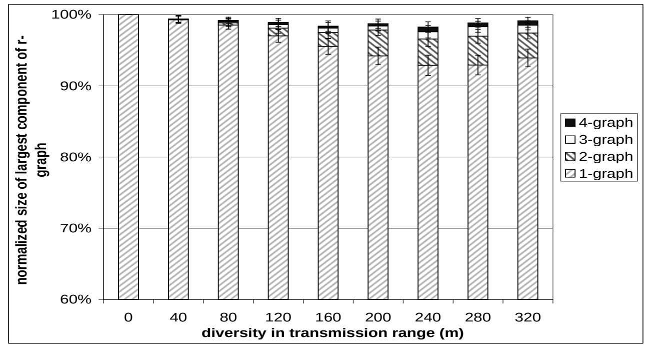


Figure 10(b): Average size of the largest connected component in r-graph. D-model, density = 50 nodes/$km^2$

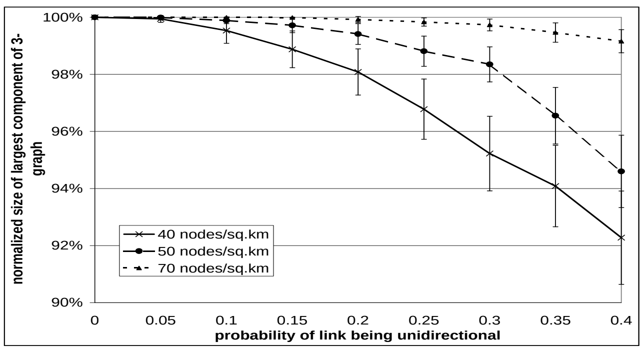


Figure 10(c): Average size of the largest connected component in 3-graph. P-model.

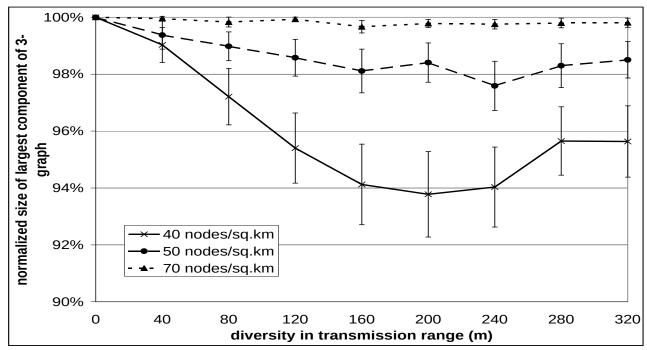


Figure 10(d): Average size of the largest connected component in 3-graph. D-model.

50 nodes/$km^2$, these values are very close to that of the $\infty$-graph (97% for D-model). This leads to an important observation that ignoring the links with long reverse routes ($> 3$) only marginally affects the connectivity of the network. Even at a low density of 40 nodes/$km^2$, the size of the largest component in the 3-graph is quite high. The normalized size of the largest component initially decreases with diversity but starts to increase at higher values of diversity. When the diversity is large the network gets more disconnected, making the size of the largest component in the $\infty$-graph small, as well as decreasing the importance of unidirectional links with long reverse routes.

### Average Distance Statistics

The graphs in Figures 11(a) and 11(b) display the average length of routes in number of hops in r-graphs for different values of $r$ in P-model and D-model at a density of 50 nodes/$km^2$ There is an increase in the average distance of the bi-graph as unidirectional links are introduced in both the models. The decrease in the average distance of the P-model at high probabilities is because of the increase in number of connected components (see Figure 5).

In both the models, the average distance of the 2-graph is lower than that of bi-graph. Including the 2-links decreases the average route length by at most 1 hop. This is because the unidirectional links can be used for transmission when there are reverse routes, avoiding the establishment of longer bidirectional routes (this would not happen with blacklist AODV). However, including r-links with $r > 2$ does not seem to make any significant difference to this metric. This is expected because the contribution of r-links to the connectivity is marginal for higher $r$. In the D-model, the average distance of the 2-graph (and 3-graph) can be seen to decrease in general as the diversity increases. This can be explained by the presence of more long-reaching nodes as the diversity increases. The nodes with greater values of transmission radius can reach further, thereby decreasing the average distance in the network. However, the bi-graph is not able to take advantage of this.

## 5 Conclusion

In this paper, we perform statistical analysis of topologies representing unidirectional ad hoc networks and quantify the impact of unidirectional links on the connectivity of the network.

The statistical analysis of the topologies generated indicate that the presence of unidirectional links could significantly affect the connectivity of the bi-graph and that most of the unidirectional links in a random graph would have short reverse routes (2-3 hops). Furthermore, the connectivity itself is a heavy tail distribution and can suddenly change from good to worse in real-life due to mobility. However, by including only

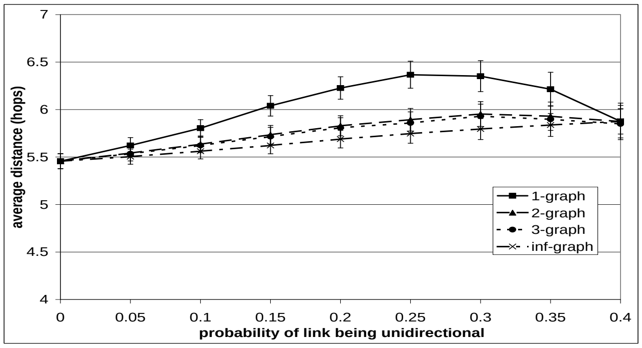


Figure 11(a): Average distance of routes in r-graph. P-model, density = 50 nodes/$km^2$

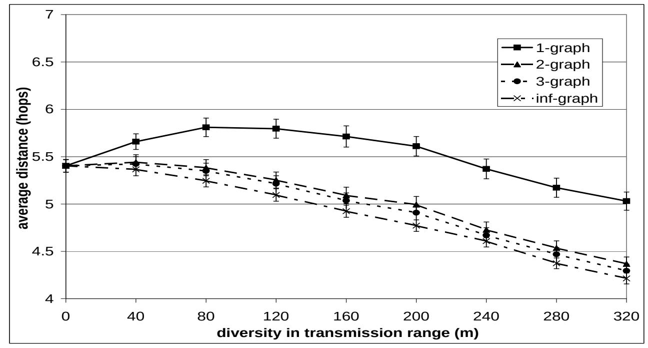


Figure 11(b): Average distance of routes in r-graph. D-model, density = 50 nodes/$km^2$

the unidirectional links with short reverse routes (2-3 hops), the connectivity can be significantly improved.

The observations stated above also provide good intuition to the developers of routing protocols. The most common approach to tackle the presence of unidirectional links has been to detect them somehow and then ignore them. Such an approach is taken by AODV [5], a popular routing protocol for ad hoc networks. Several methods to enable AODV to avoid unidirectional links are described in [4]. However, a very high density of nodes (300 nodes/$km^2$ in [4]) may be required to achieve good bidirectional connectivity in the presence of unidirectional links. Other protocols such as DSR [3] are designed to route packets even along the unidirectional links. However, they incur the expenses of maintaining long reverse routes for some of these unidirectional links. The observations in this paper suggest that a more efficient routing protocol would only employ unidirectional links with short reverse routes, as in SRL [8], and only when really required to use them (this dynamic SRL scheme is the focus of our current research).

The important observations from the analysis in this paper provides good intuition for the developers and deployers of ad hoc networks. The topologies analyzed in this paper have been randomly drawn by changing various parameters in a wide range. These topologies represent instantaneous snapshots of the network and the variation with time in the mobile network can be approximated by a series of snapshots. Thus the observations made in this paper apply to a wide variety of scenarios encountered in mobile ad hoc networks.

## References

[1] Lichun Bao and J.J. Garcia-Luna-Aceves. “Link-state Routing in Networks with Unidirectional Links”. *Eighth Internation Conference on Computer Communications and Network, ICCCN*, 1999.

[2] Deepak Ganesan, Bhaskar Krishnamachari, Alec Woo, David Culler, Deborah Estrin, and Stephen Wicker. An Empirical Study of Epidemic Algorithms in Large Scale Multihop Wireless Networks, *UCLA Computer Science Technical Report UCLA/CSD-TR 02-0013*.

[3] D. B. Johnson, and D. A. Maltz. “Dynamic Source Routing in Ad-Hoc Wireless Networks”. *Mobile Computing*, edited by T. Imielinski and H. Korth, chapter 5, pp. 153-181, Kluwer, 1996.

[4] Mahesh Marina, and Samir R. Das. “Routing Performance in the Presence of Unidirectional Links in Multihop Wireless Networs.” *In Proceedings of MOBIHOC 2002*, Lausanne, Switzerland, June 9 - 11, 2002. (to appear)

[5] C. Perkins, E. Royer, S. Das “Ad-Hoc On Demand Distance Vector(AODV) Routing”. *IETF Internet Draft, draft-ietf-manet-aodv-08.txt*, March, 2001. Work in Progress.

[6] R. Prakash. “Unidirectional Links Prove Costly in Wireless Ad-Hoc Networks”. *Proceedings of ACM DIAL M’99 Workshop*, Seattle, WA, August 1999, pp. 15-22.

[7] Elizabeth M. Royer and Charles E. Perkins. “Ad-hoc On-Demand Distance Vector Routing”. *Proceedings of the 2nd IEEE Workshop on Mobile Computing Systems and Applications*, Feb. 1999, 90-100.

[8] Venugopalan Ramasubramanian, Ranveer Chandra, and Daniel Mossé. “Providing a bidirectional abstraction for unidirectional ad hoc networks”. *In Proceedings of IEEE INFOCOM*, June 2002.

[9] Xiang Zeng, Rajive Bagrodia, and Mario Gerla. “GloMoSim: A Library for Parallel Simulation of Large-scale Wireless Networks”. *Proceedings of the 12th workshop on Parallel and distributed simulation*, May 26 - 29, 1998, Banff Canada, 154-161.