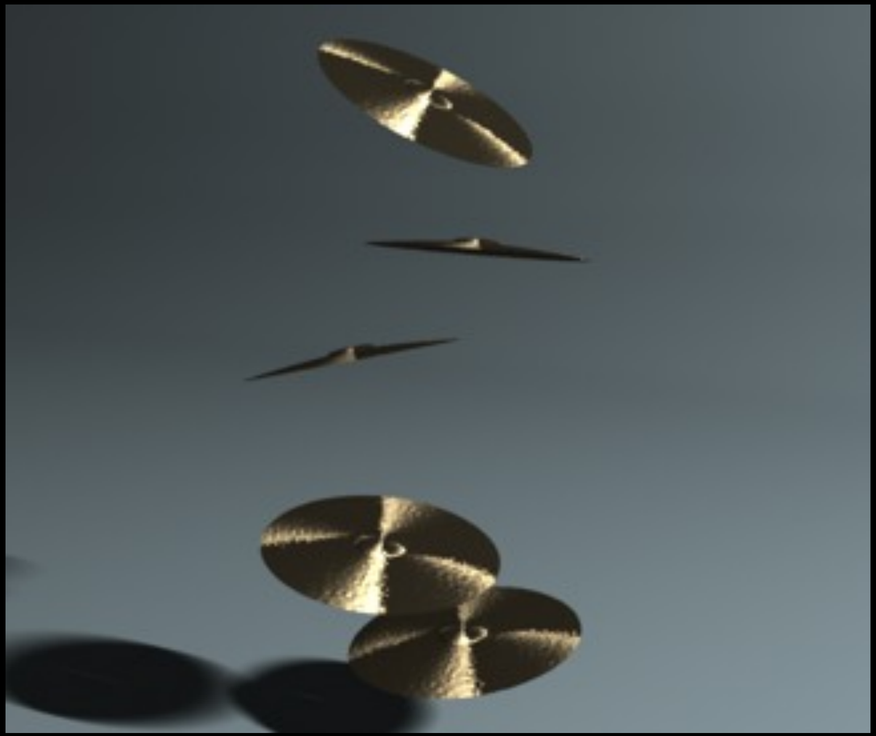
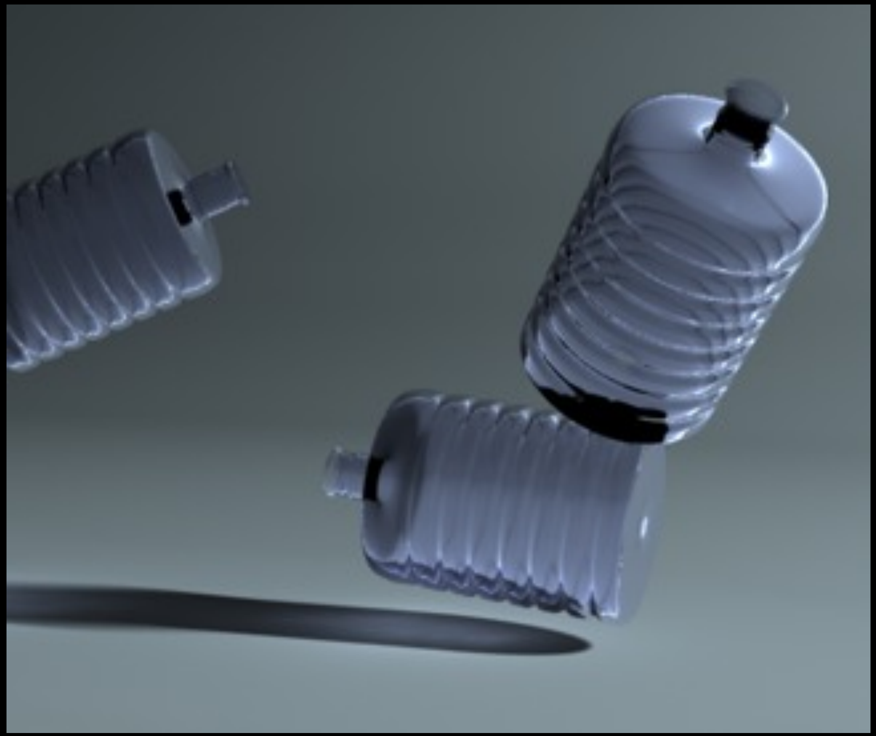
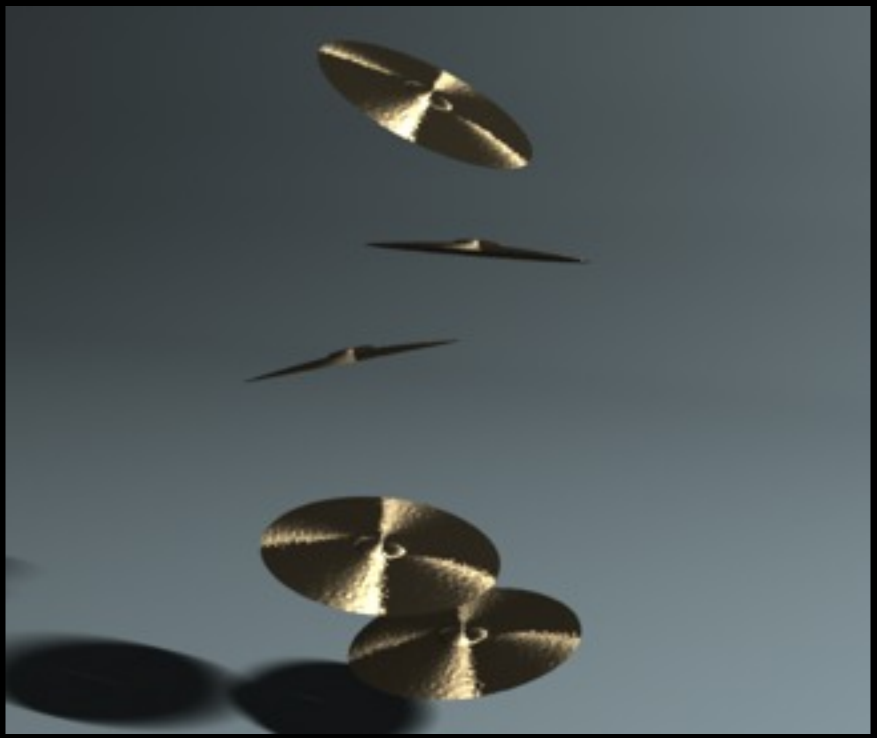
Harmonic Shells

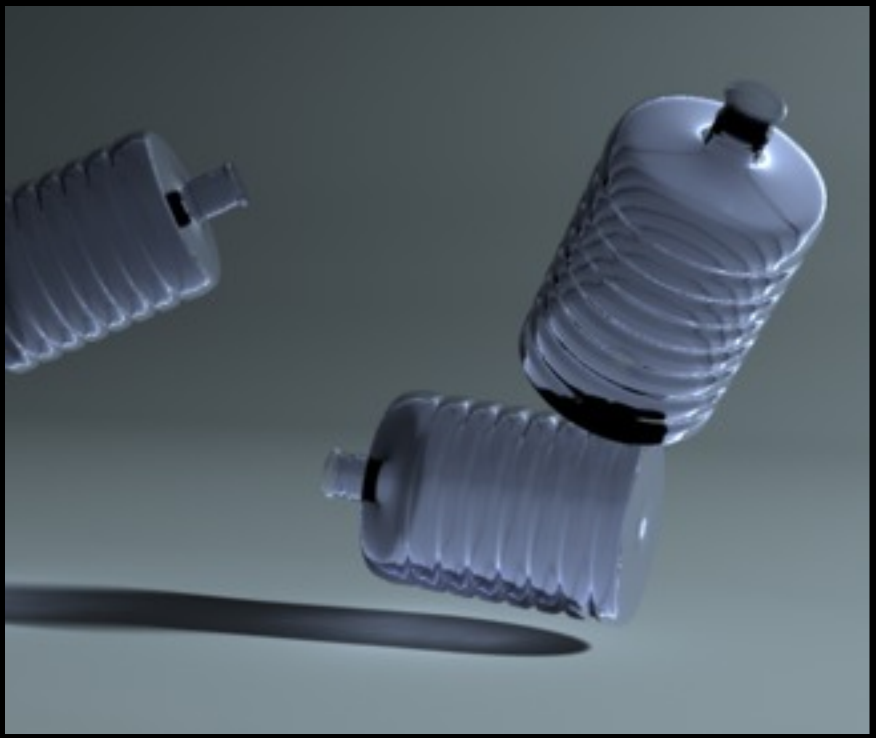
A Practical Nonlinear Sound Model for Near-Rigid Thin Shells

Jeffrey Chadwick, Steven An and Doug James

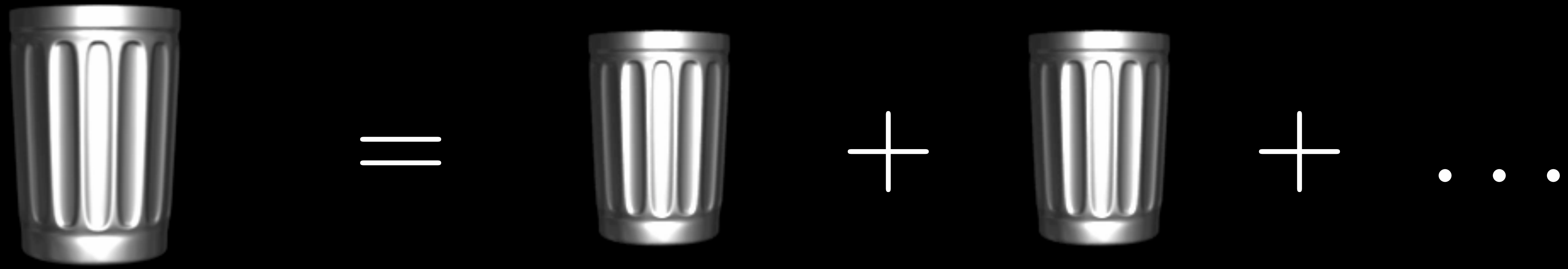
Cornell University





Linear Modal Sound Synthesis



Linear modal sound

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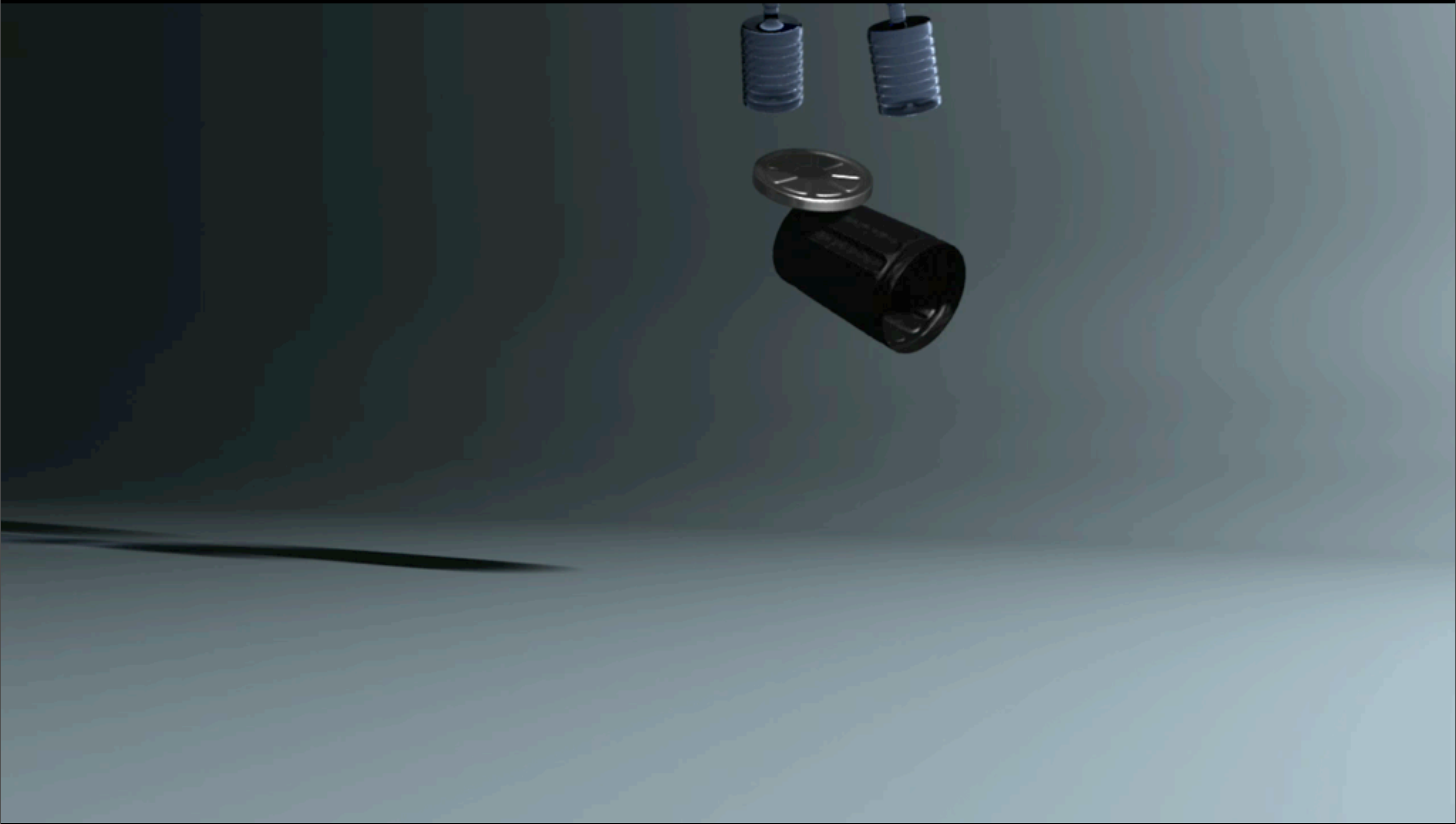

Linear modal sound + transfer

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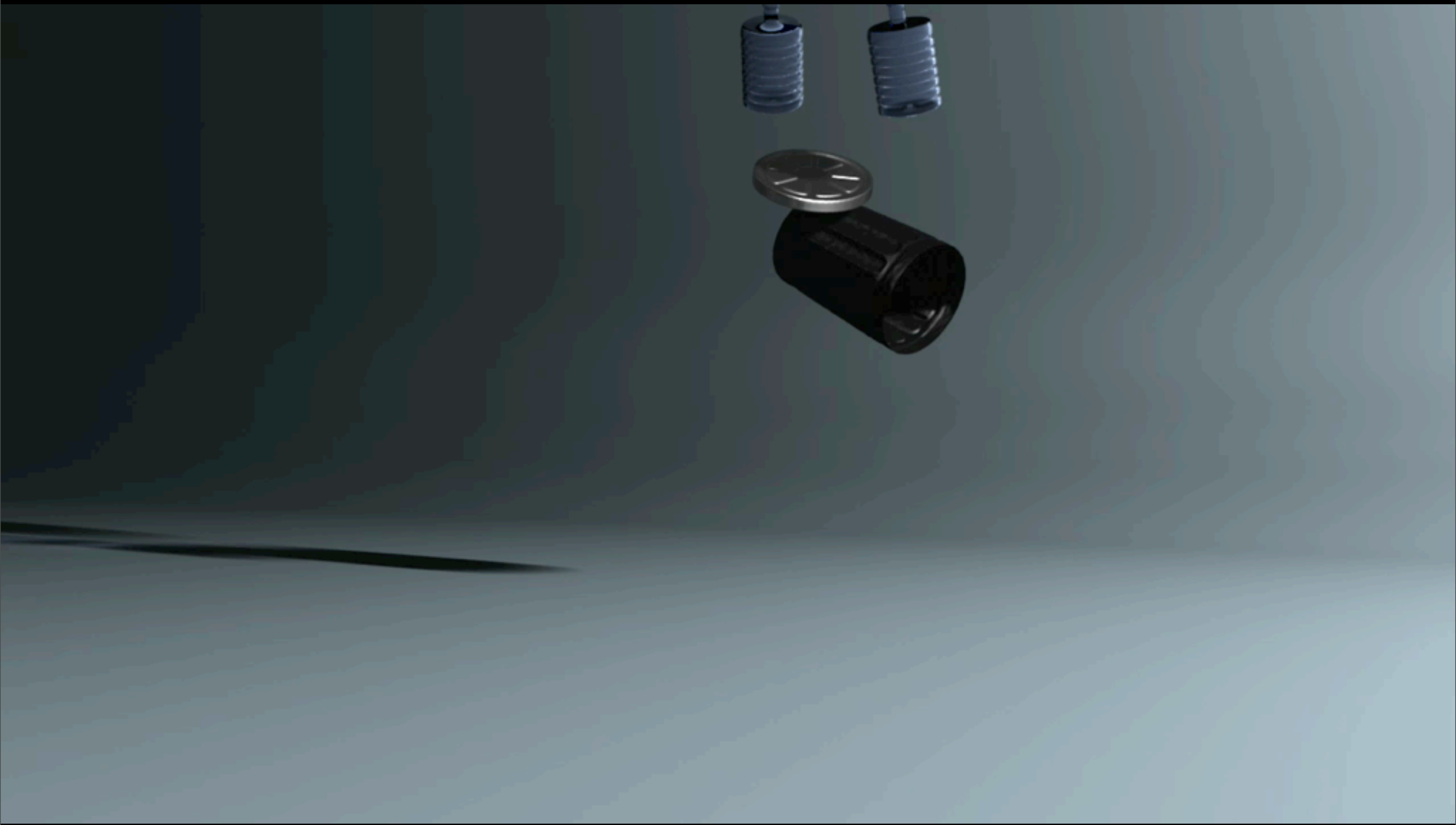

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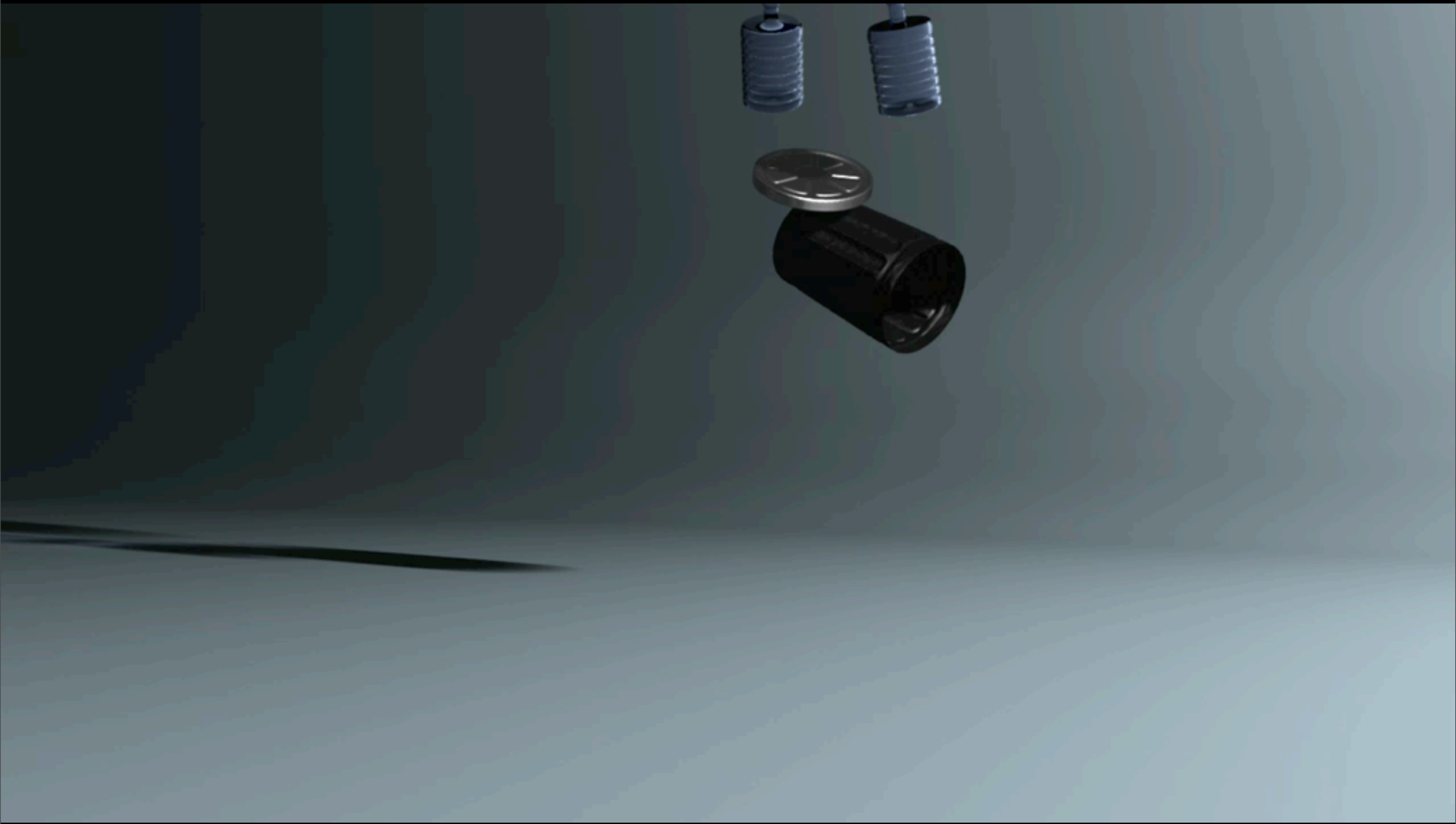

Motivation

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Rigid objects: vibrations approximated well by linear dynamics

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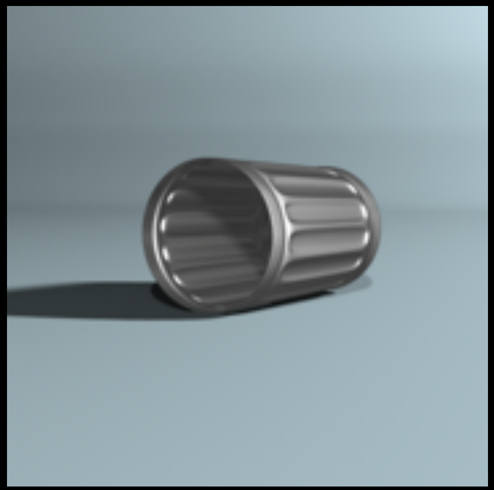
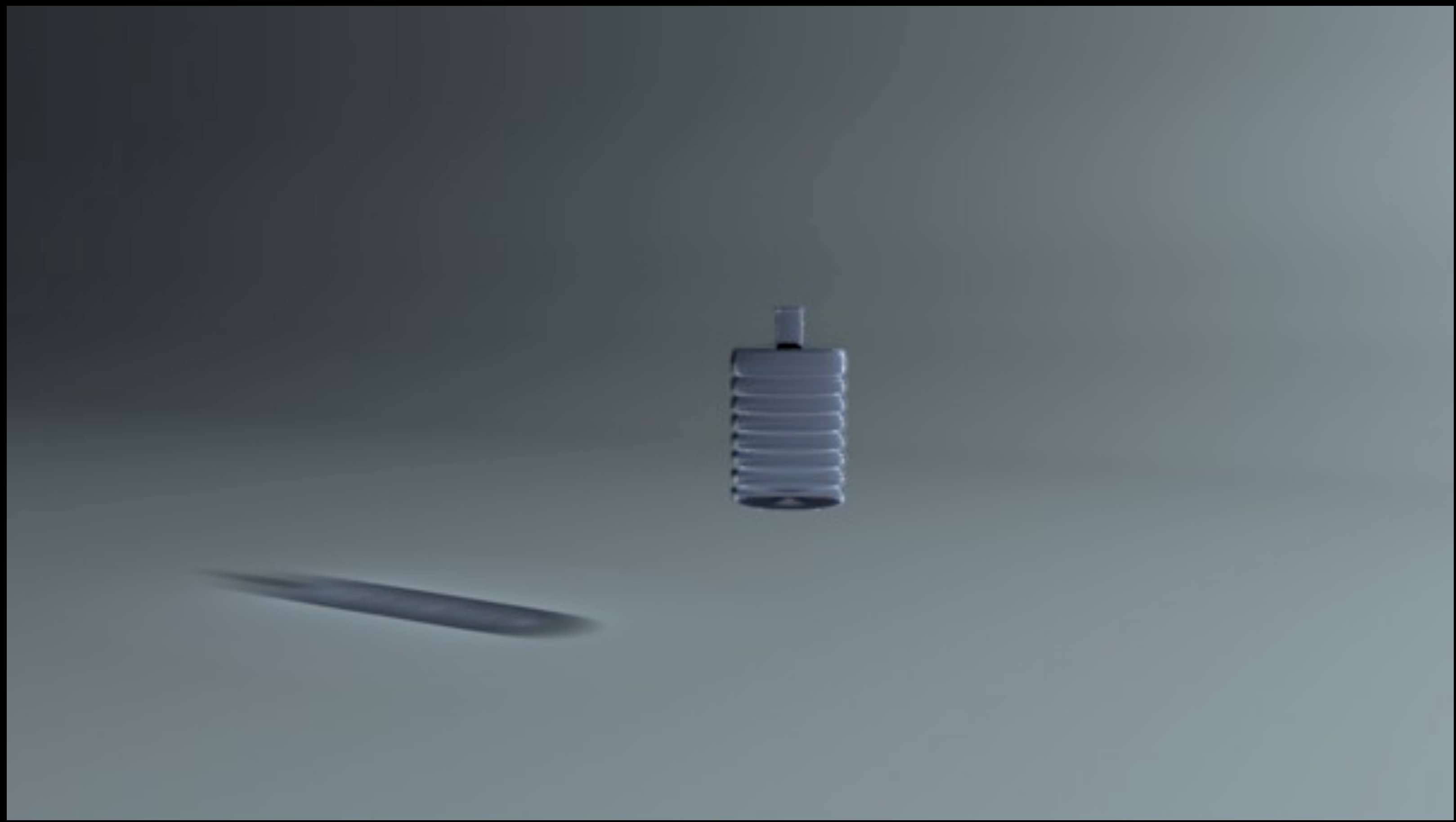


Shell structures: exhibit noisy nonlinear behavior (even under modest forcing)

Motivation

Linear modal sound simulation (side impact):



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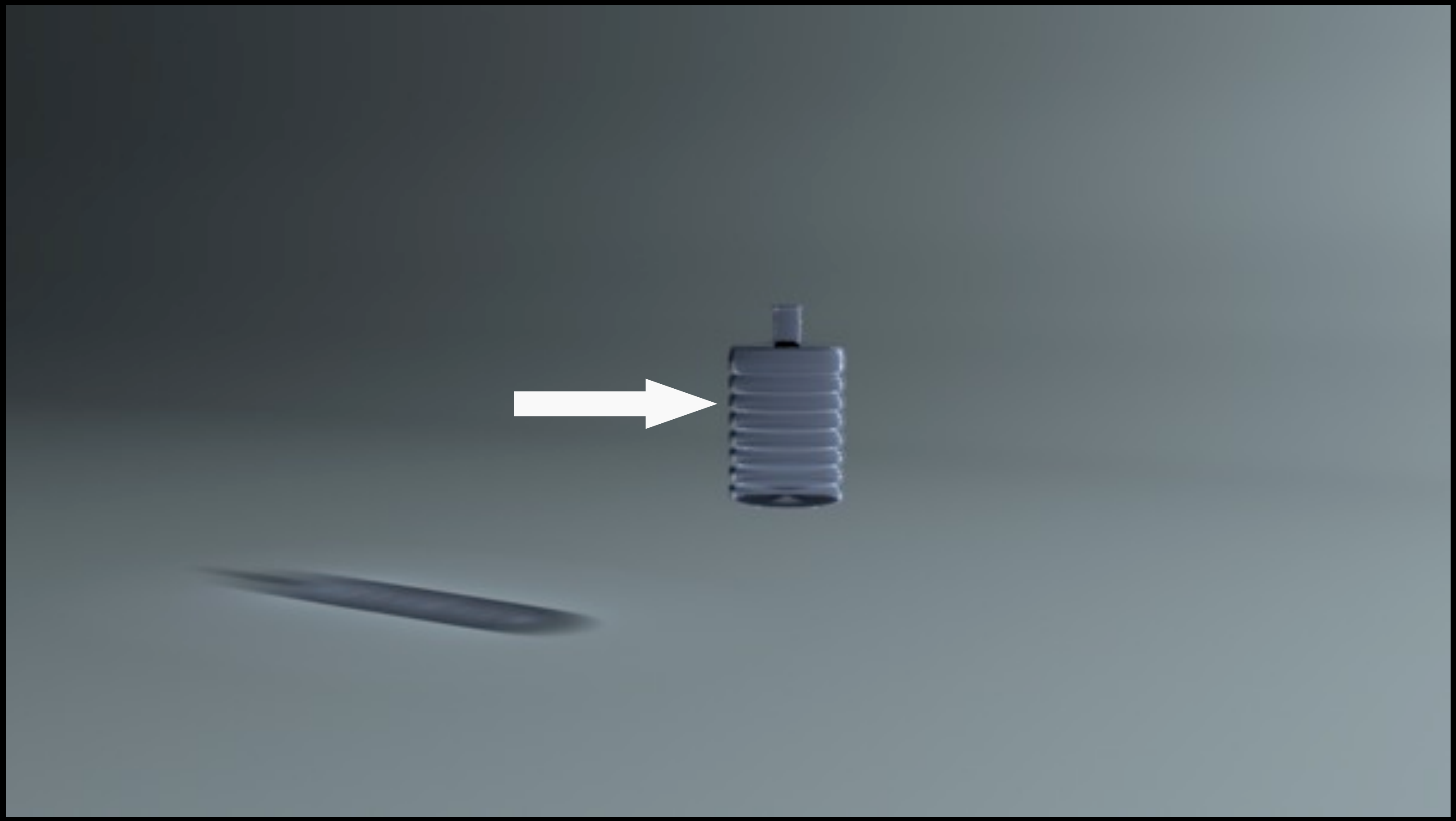

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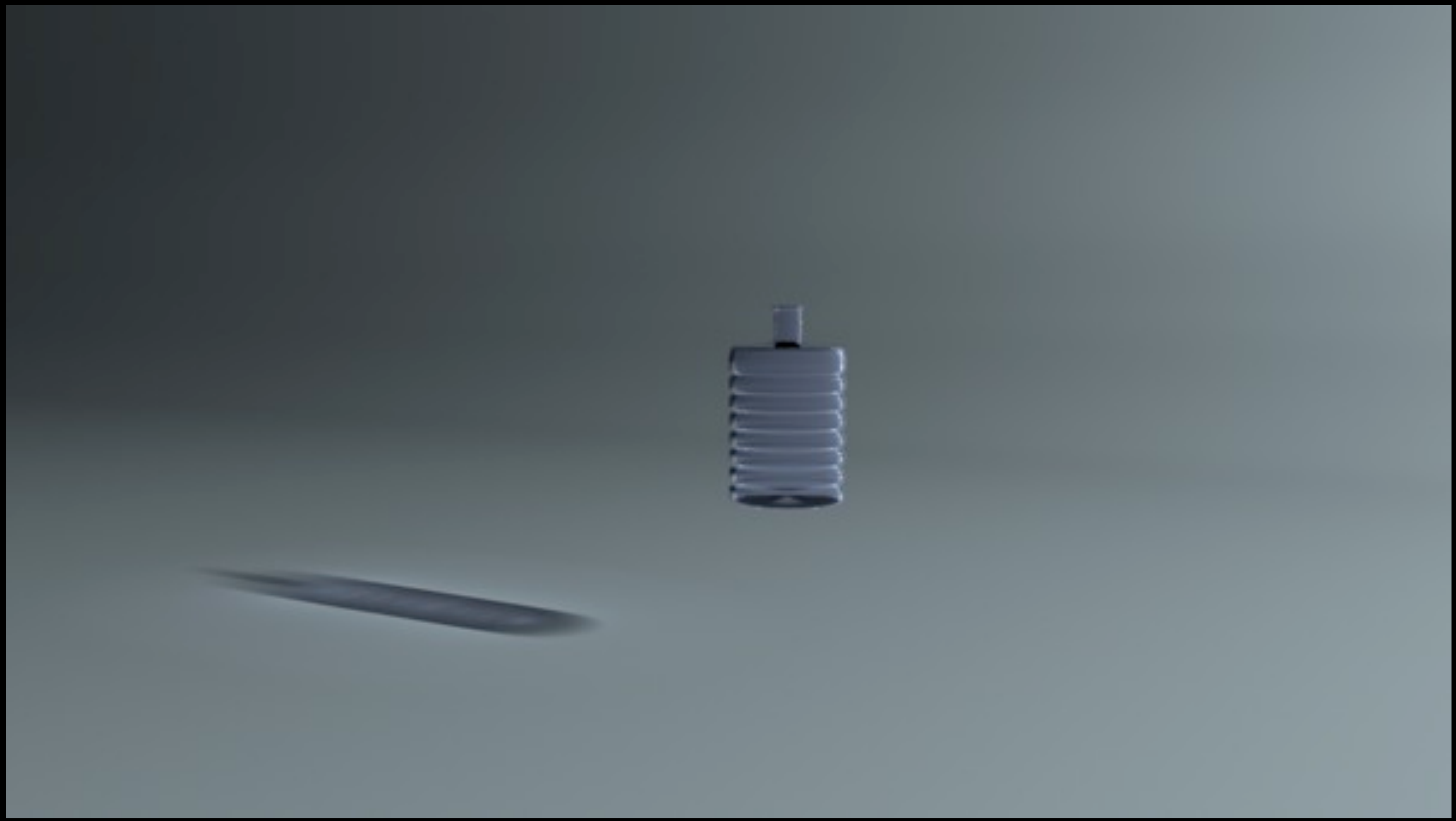

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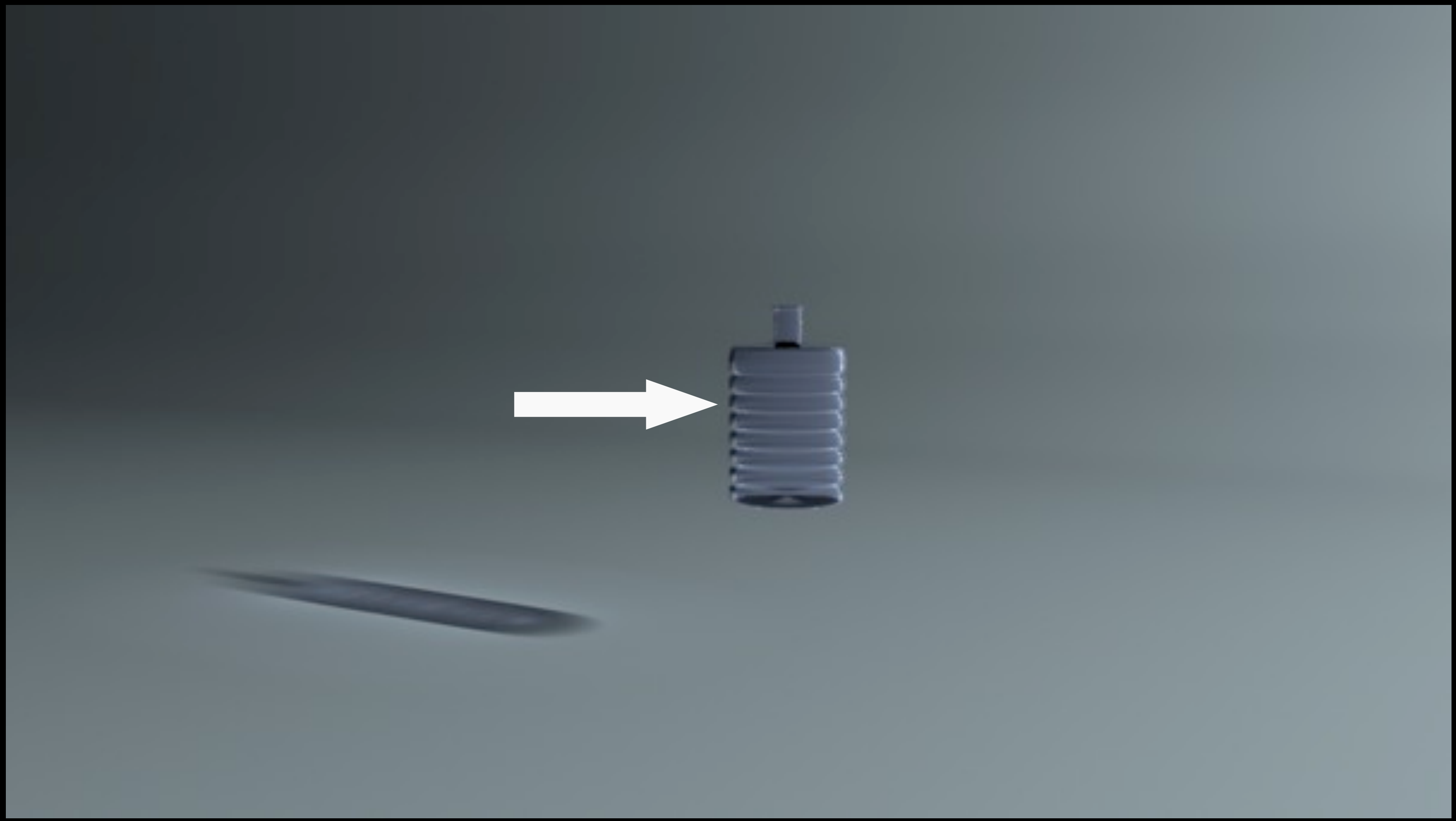

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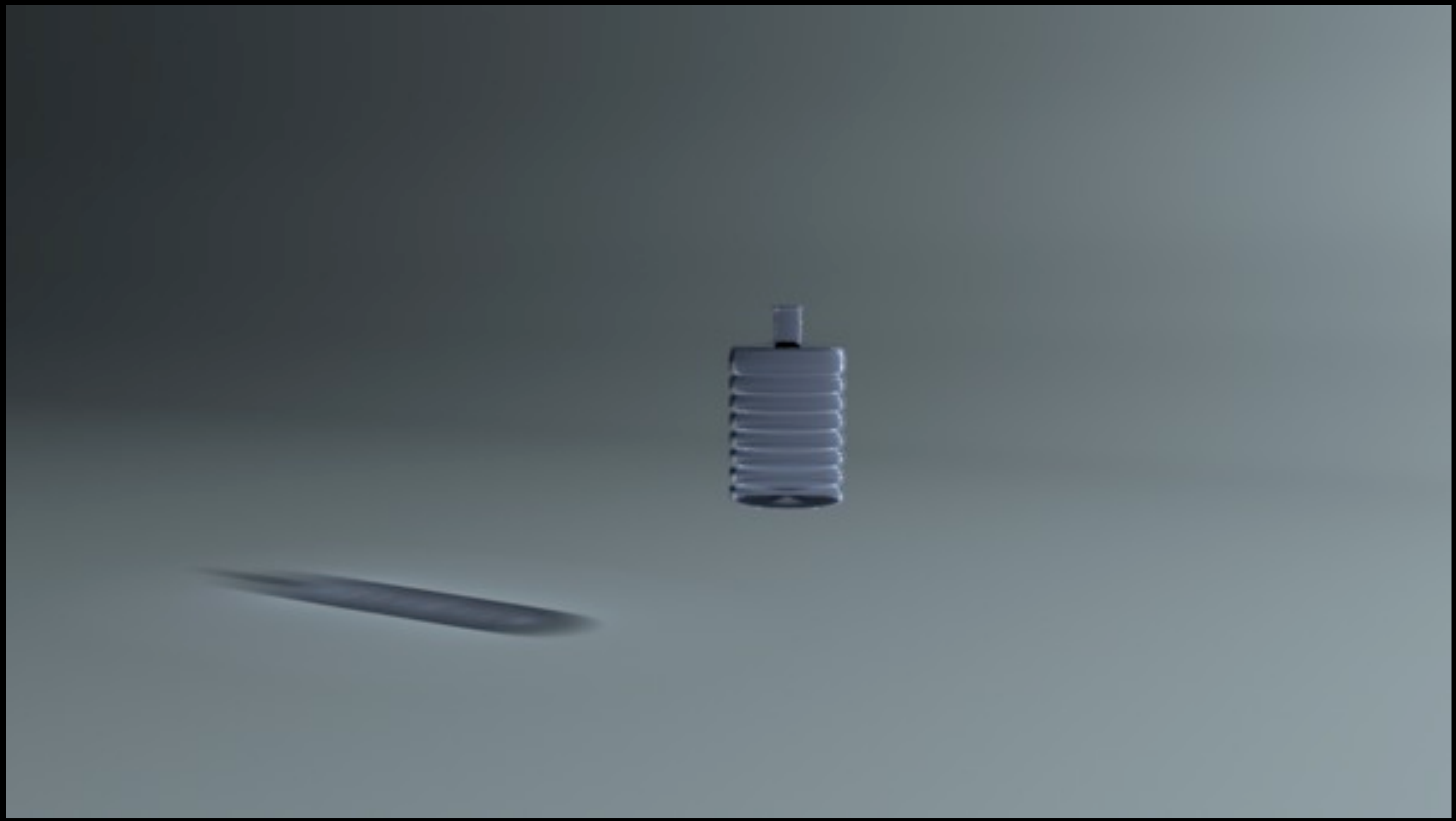

(... but this took about 19 days to synthesize)

Harmonic Shells

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- A practical approach to computing nonlinear vibrations for thin shells

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- Extend standard linear modal sounds by introducing nonlinear mode coupling and force response

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Harmonic Shells

- A practical approach to computing nonlinear vibrations for thin shells
- Extend standard linear modal sounds by introducing nonlinear mode coupling and force response
  - Richer sounds than linear models
- A texture-based method for fast (O(1) per mode) acoustic transfer computation

Related Work

Linear Modal Sounds

Linear Modal Sounds:
eg. [van den Doel et al. 1996]

Frequently used in graphics, eg:

"FoleyAutomatic"
[van den Doel et al. 2001]

"Synthesizing Sounds from Rigid-Body Simulations"
[O'Brien et al. 2002]

[O'Brien et al. 2002]

[Boneel et al. 2008]

Related Work

Linear Modal Sounds

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- Fails to capture a lot of interesting sound behavior

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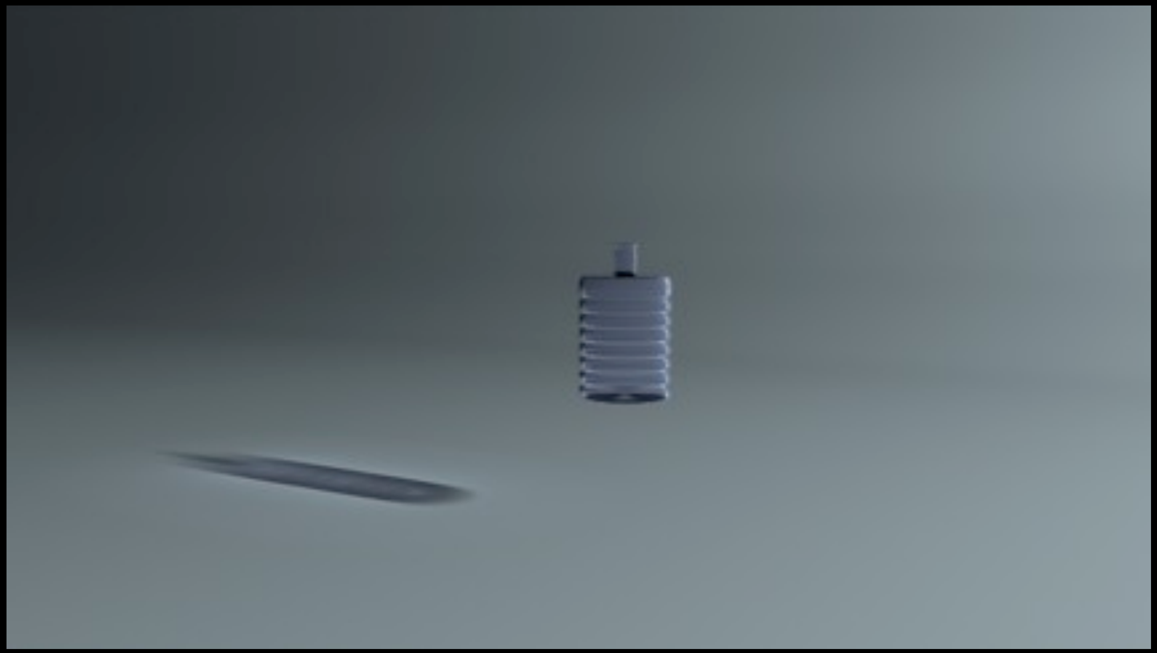
- Simple example: sound characteristics (not just volume) change with impact magnitude



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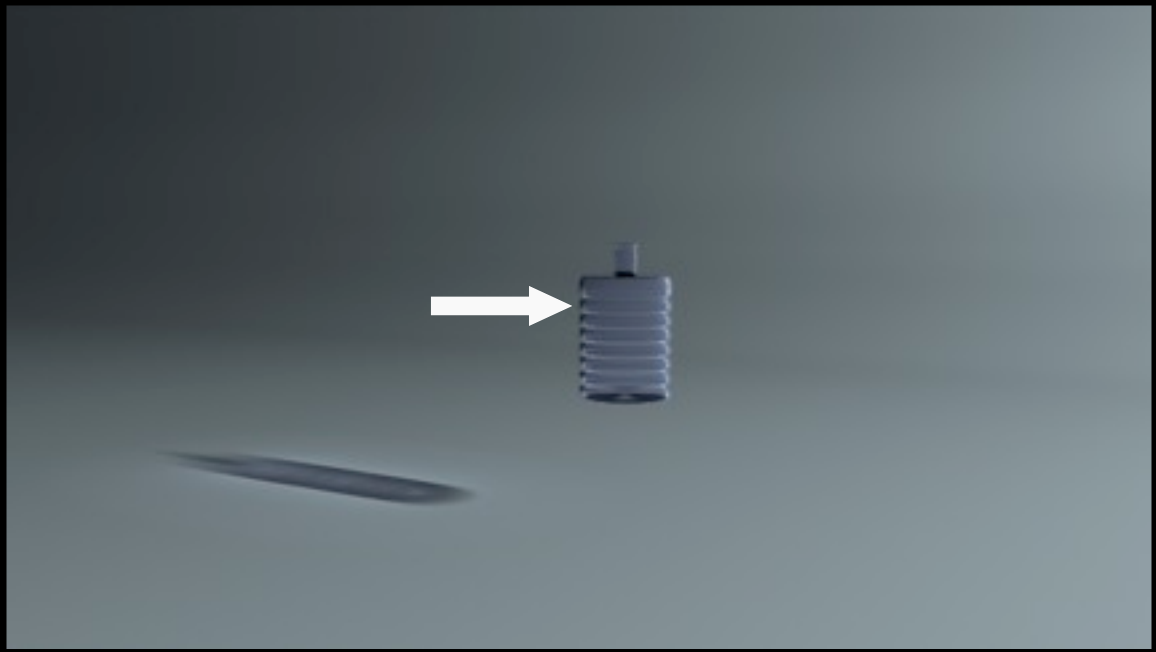

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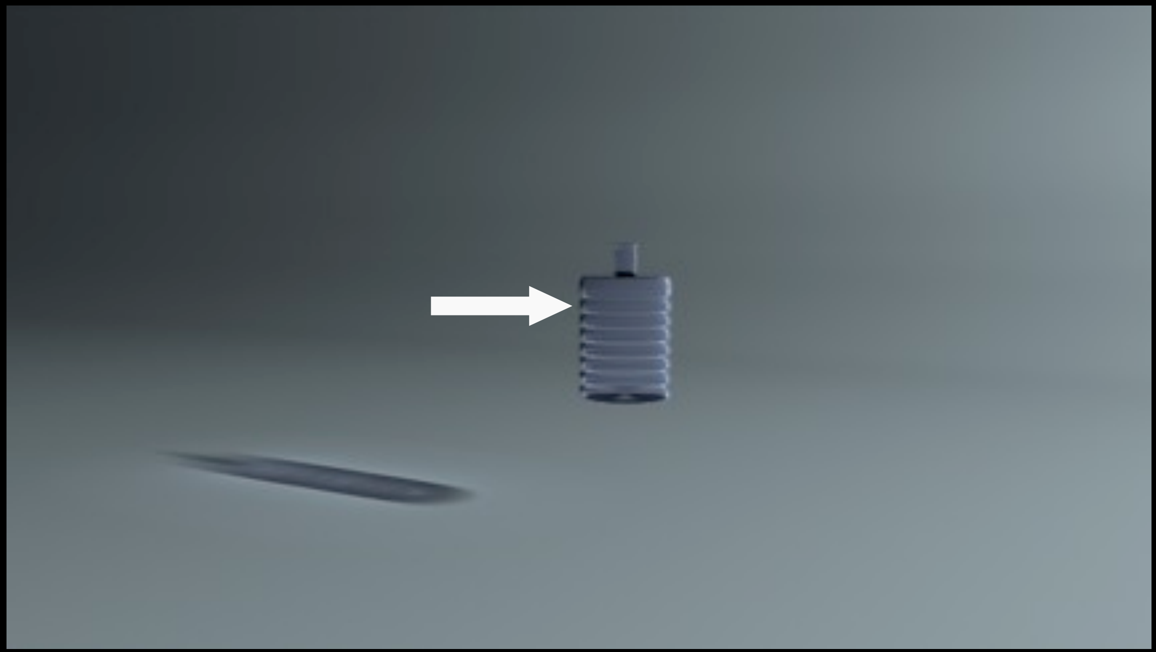

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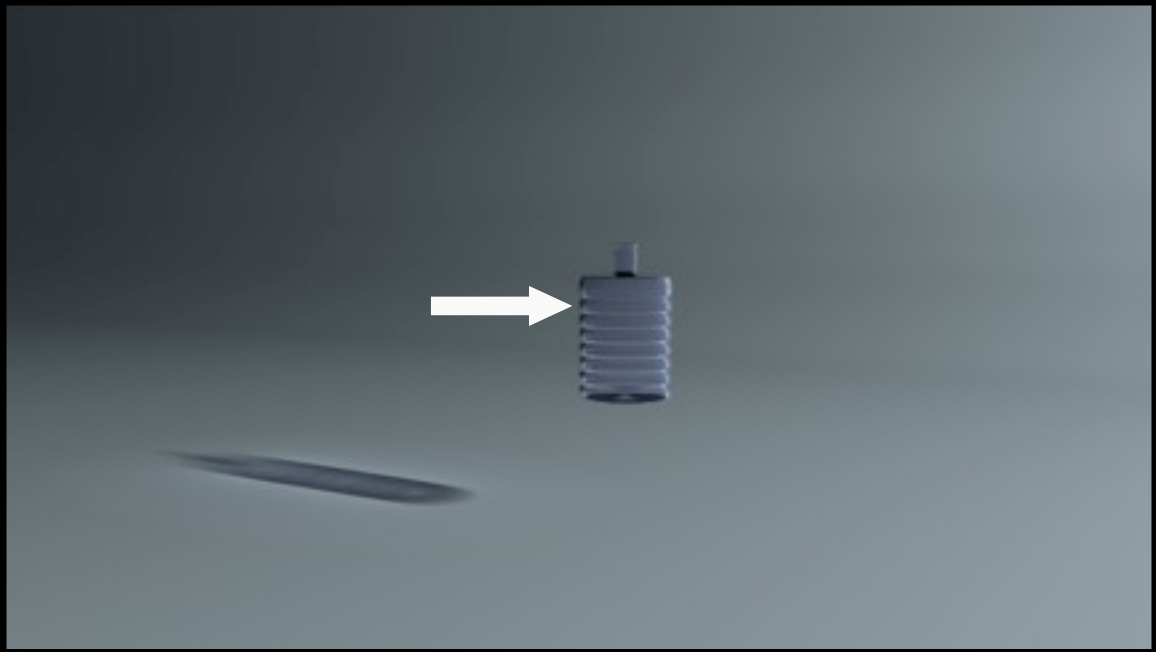

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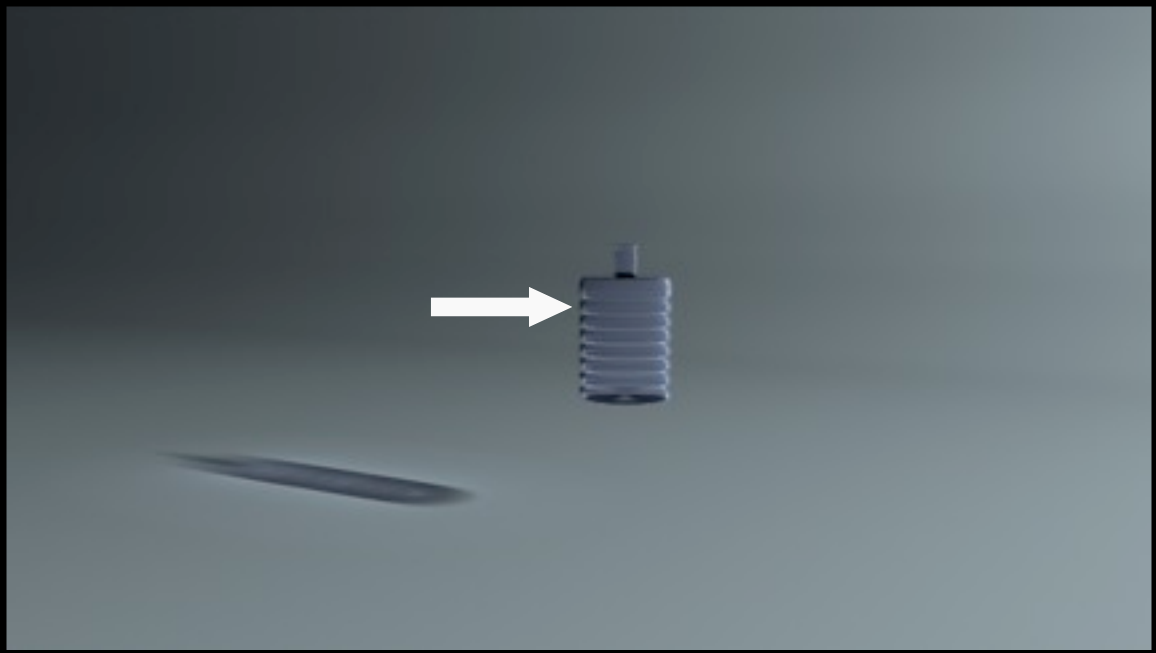

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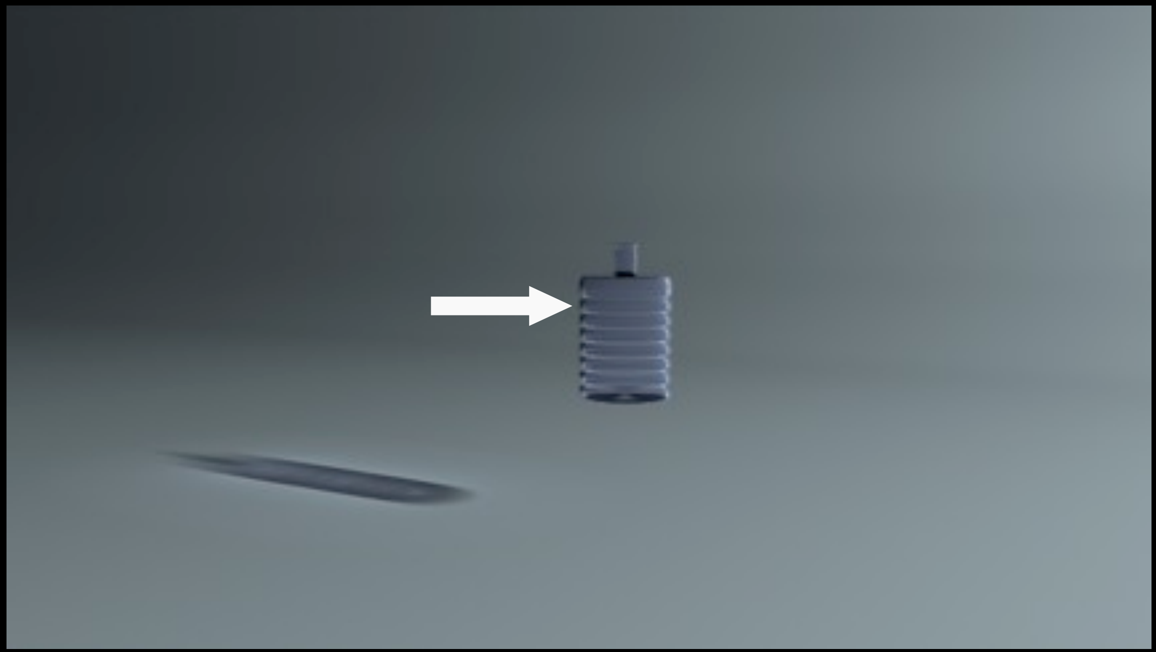
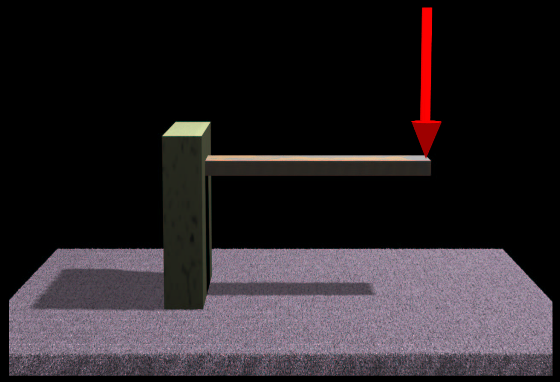


  - Linear model does not capture this

Related Work

Nonlinear vibrations and sound

"Synthesizing Sounds from Physically Based Motion"
[O'Brien et al. 2001]



[O'Brien et al. 2001]

Efficient, conservative numerical schemes for nonlinear plates and strings
[Bilbao 2005, 2008]

"Nonlinear vibrations and chaos in gongs and cymbals"
[Chaigne et al. 2005]

No efficient nonlinear synthesis methods for sound in animation

Algorithm Overview

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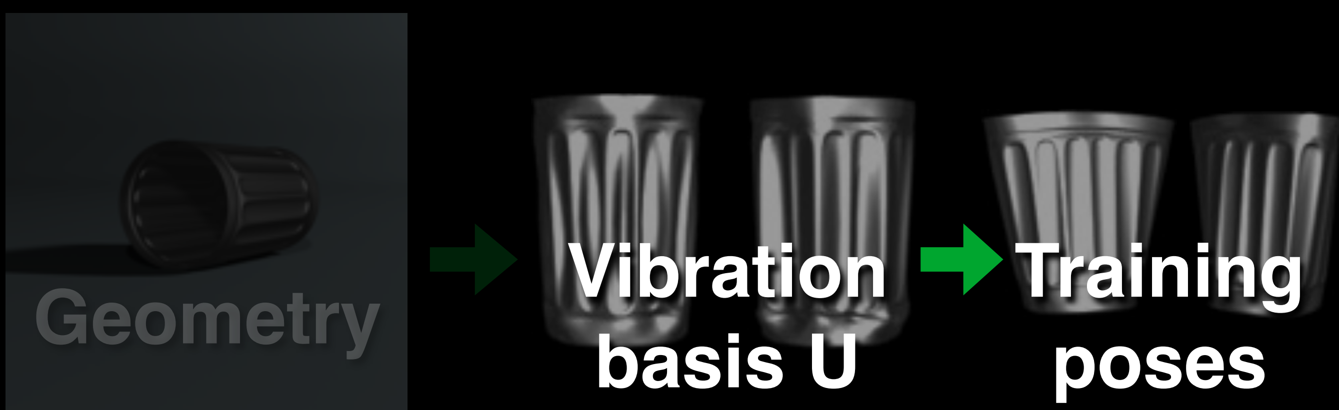

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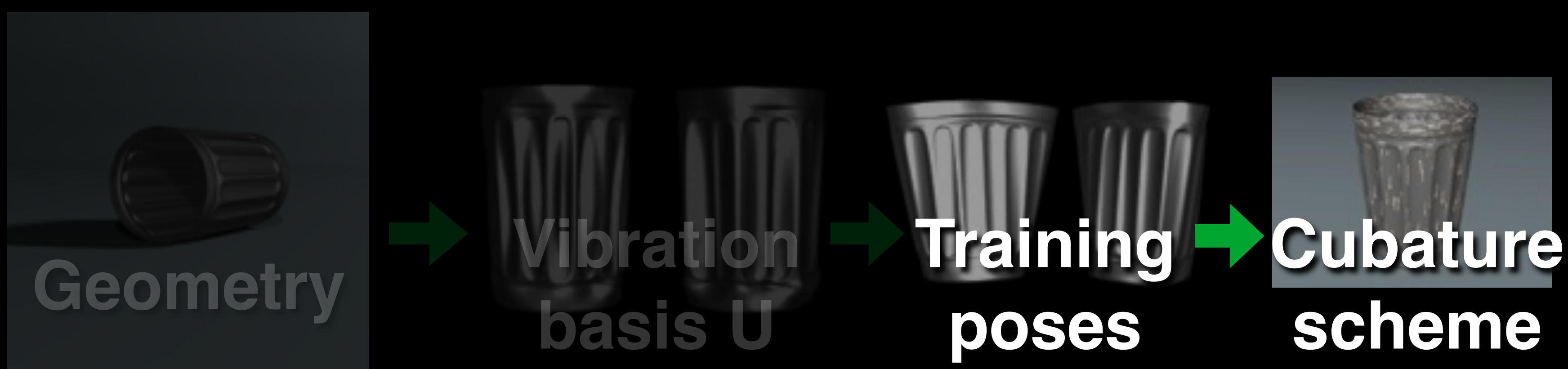

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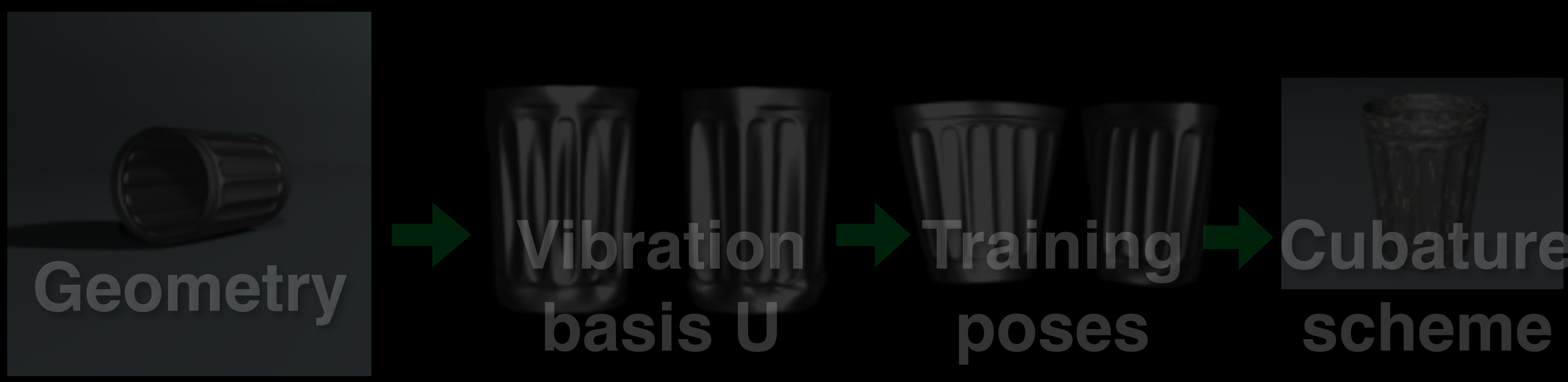

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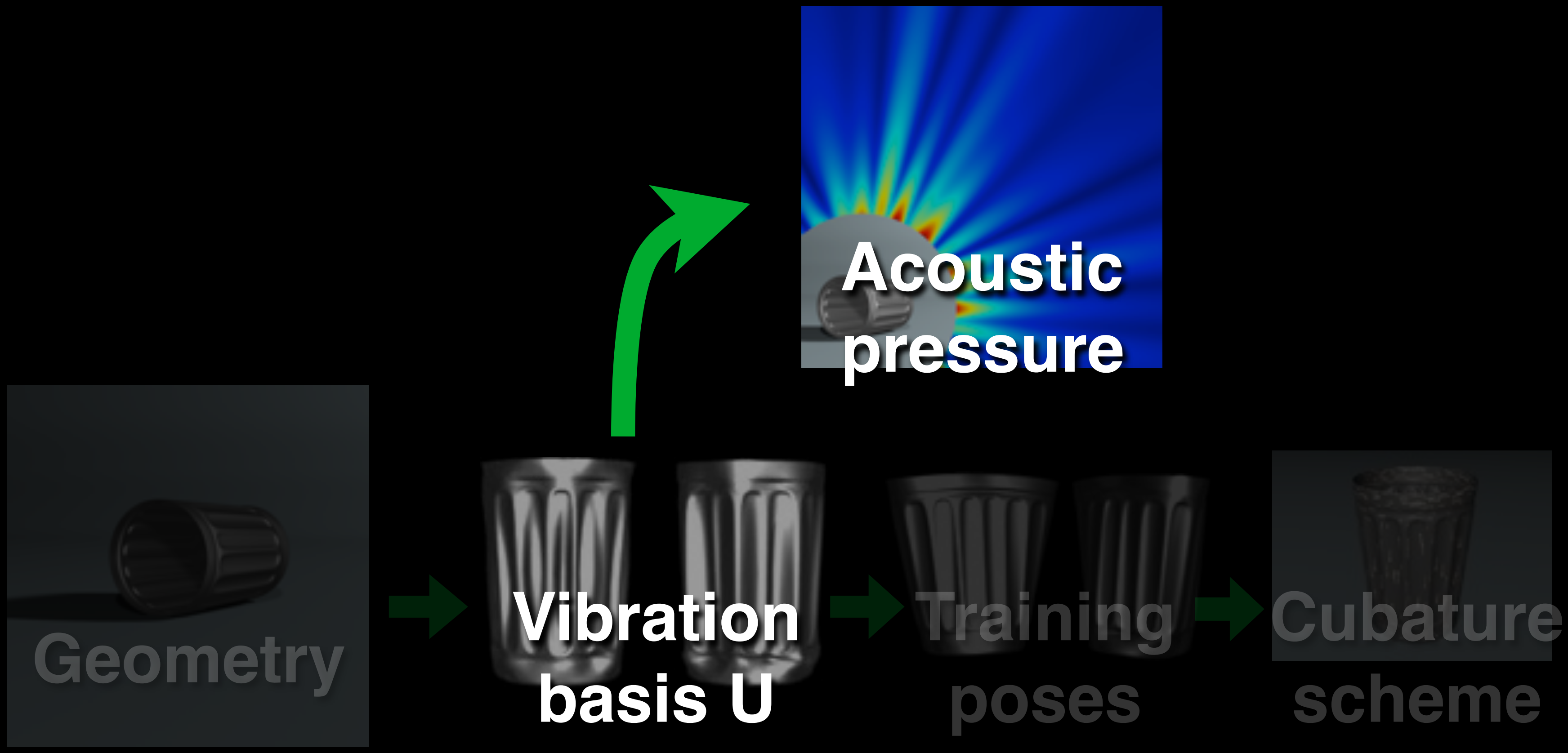

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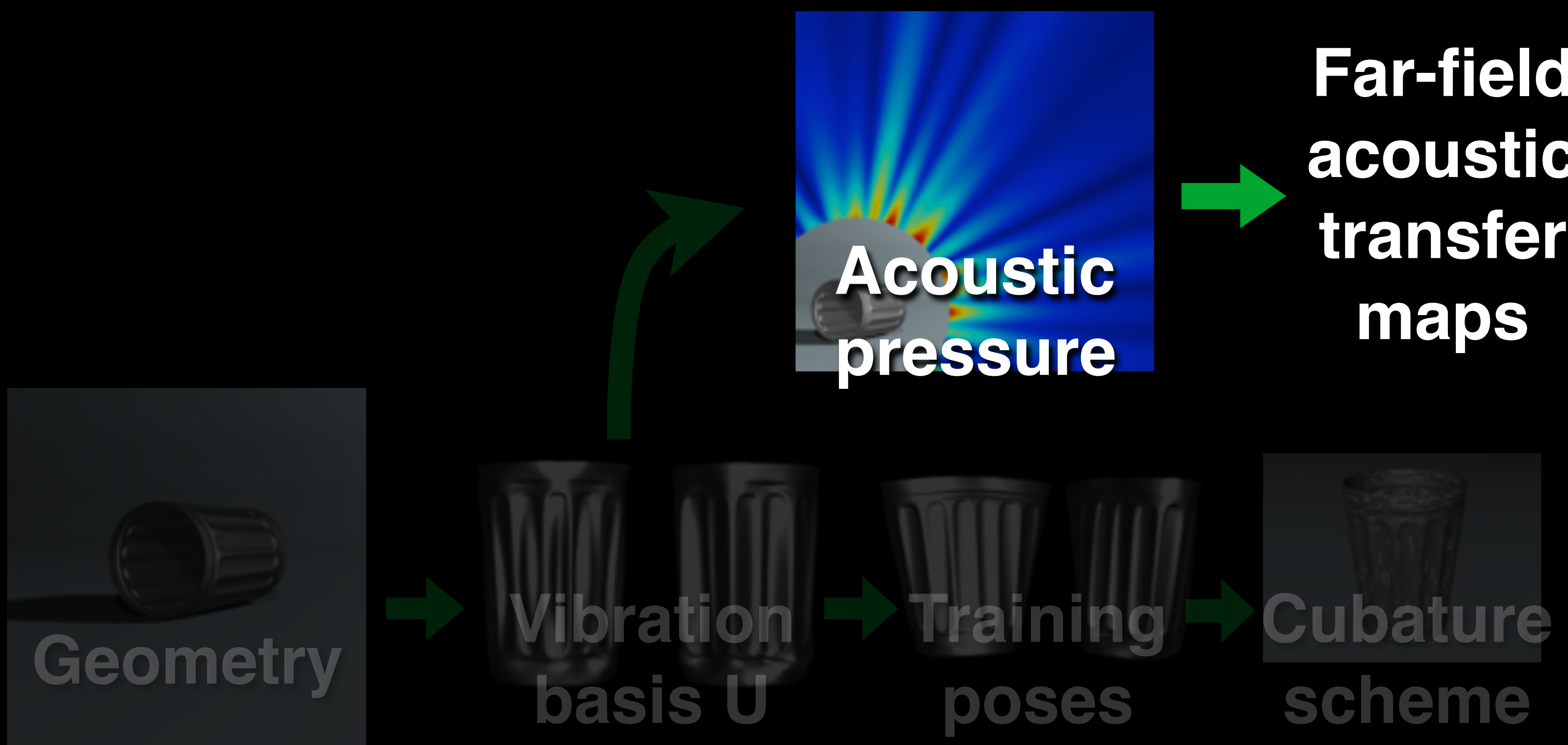

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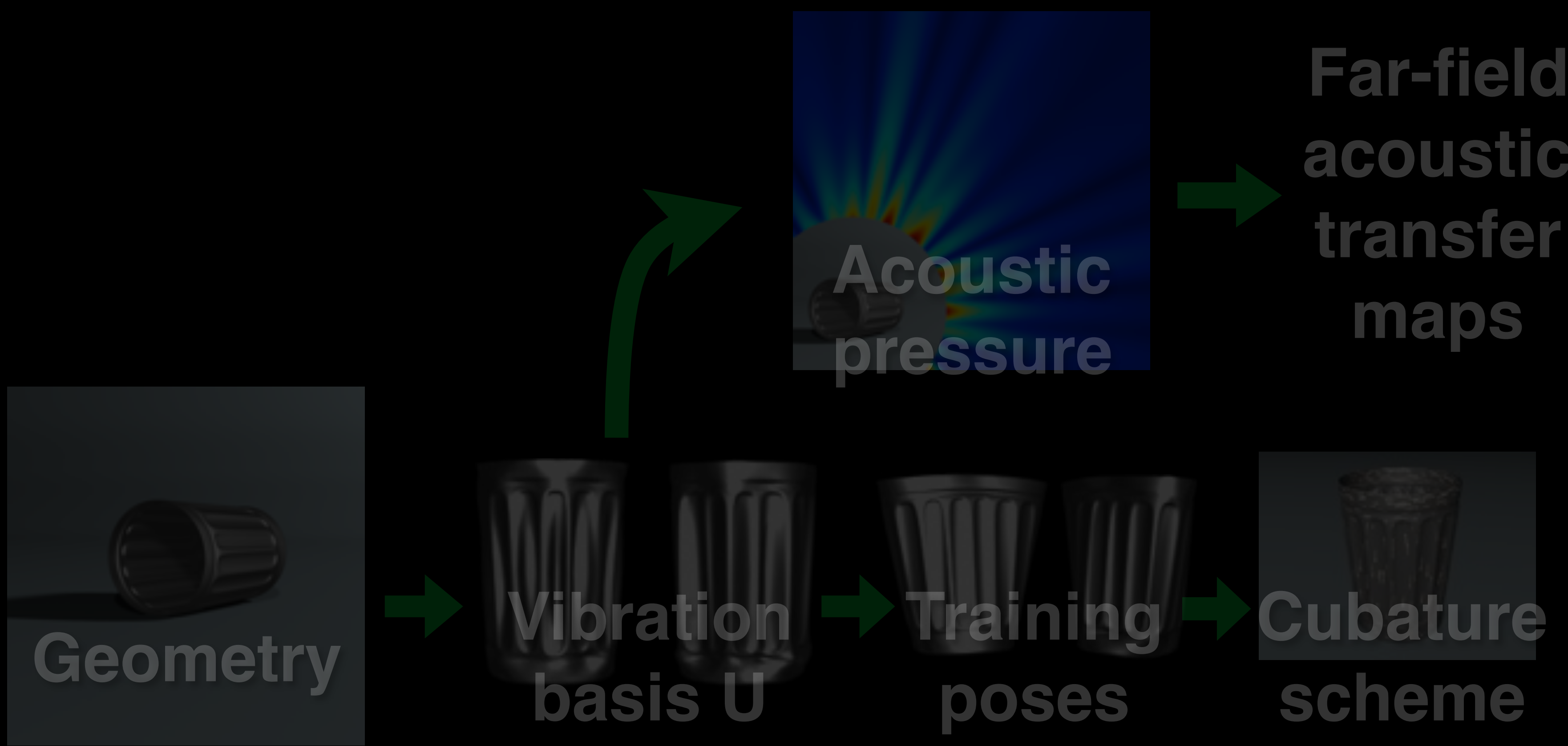

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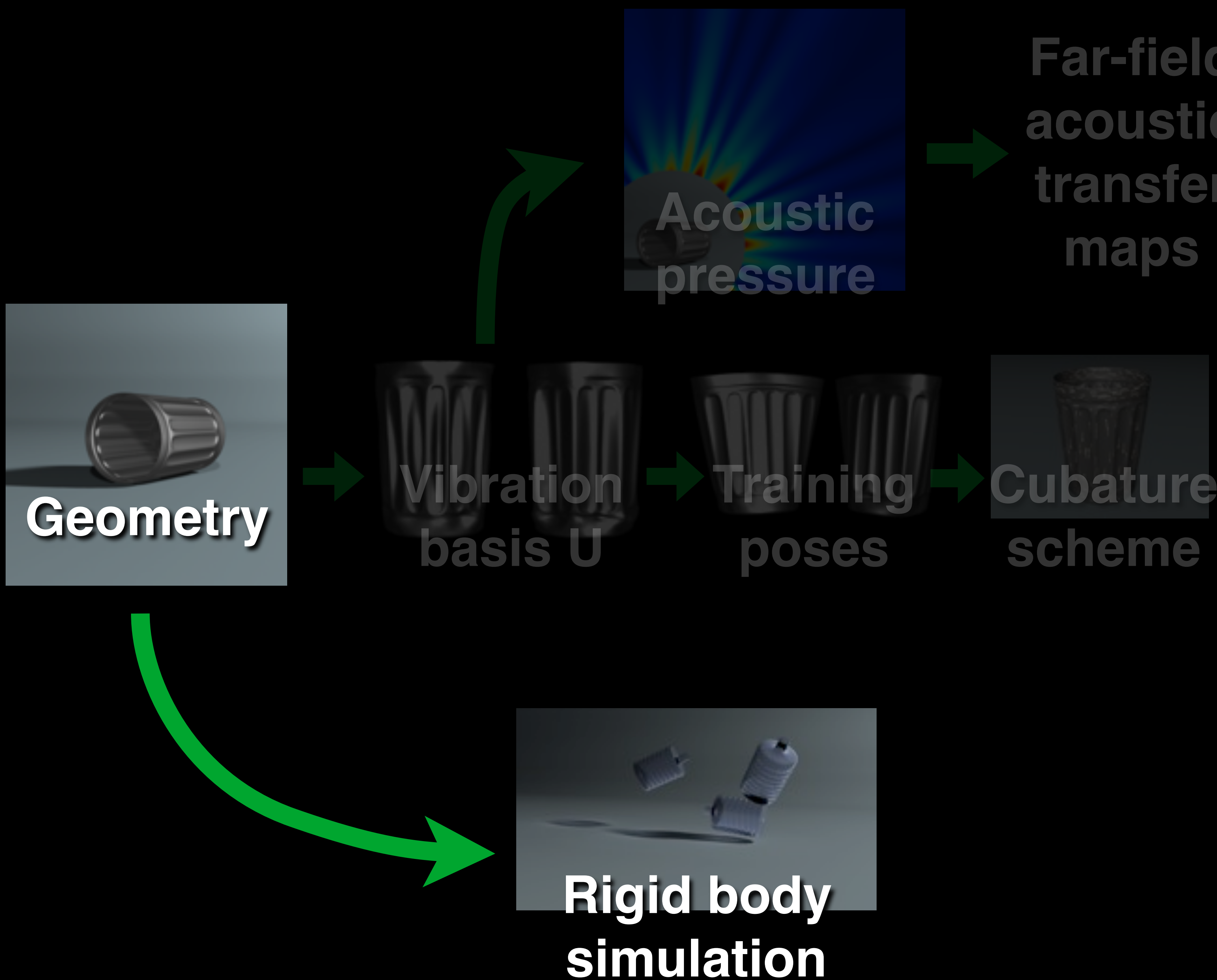

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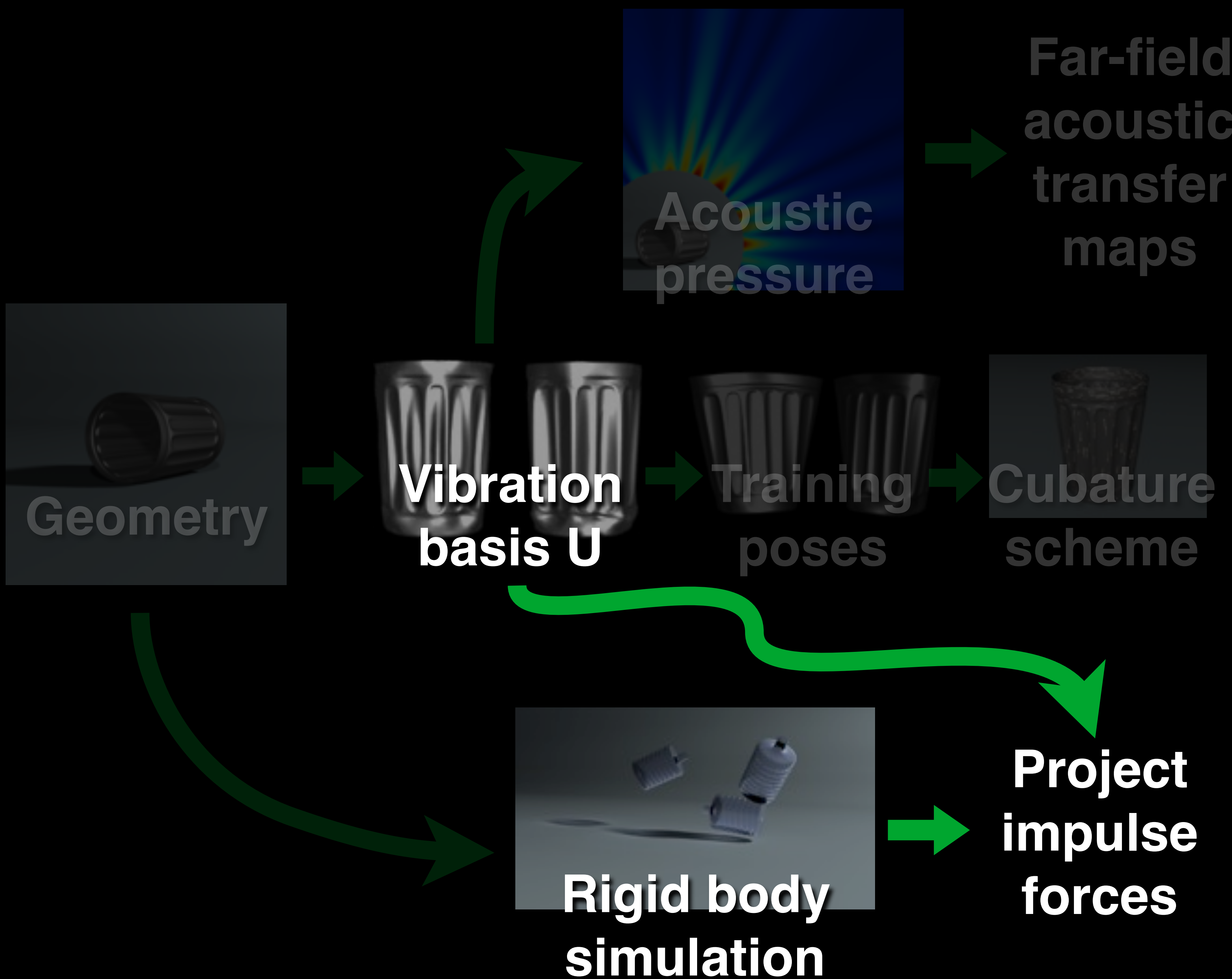

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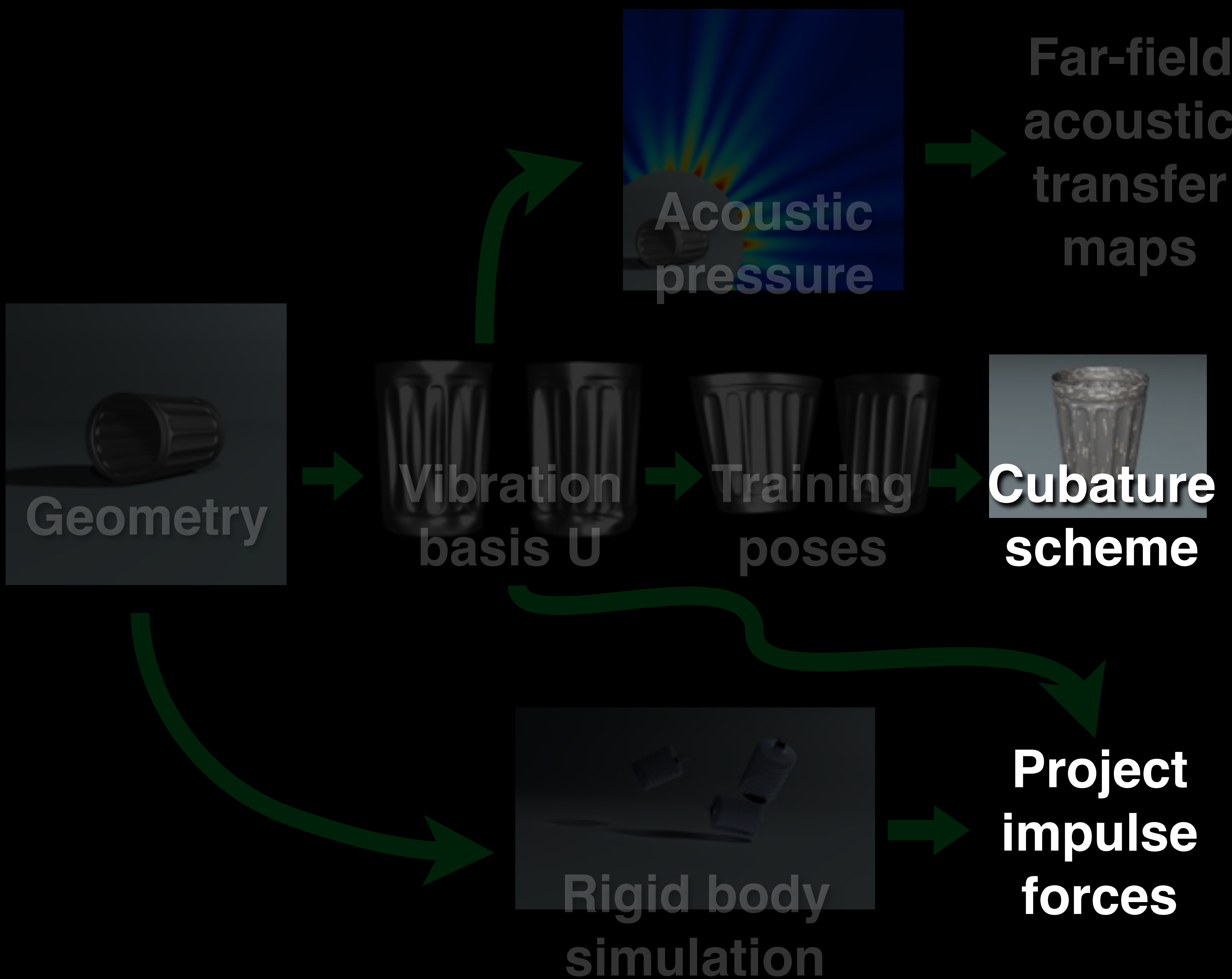

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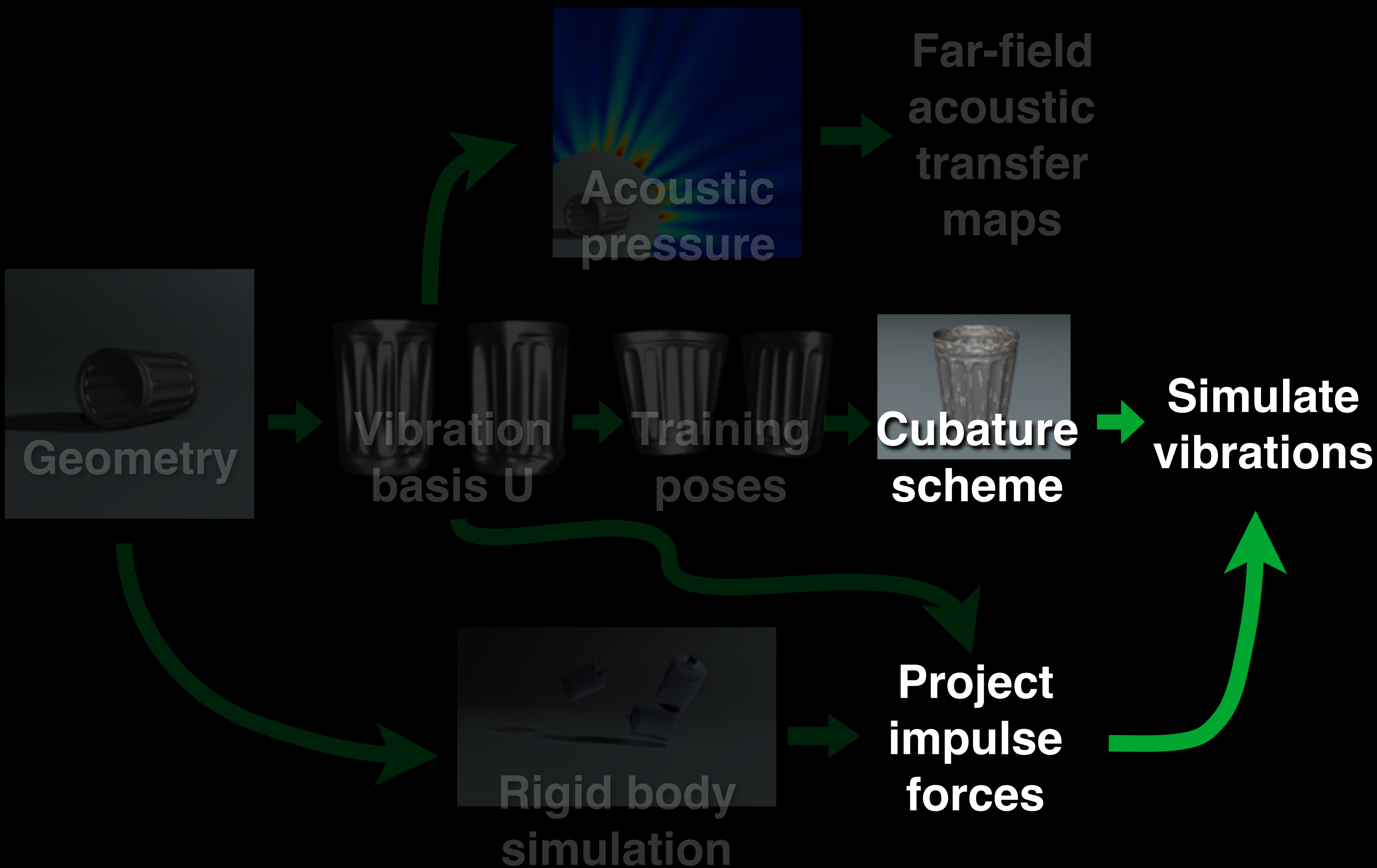

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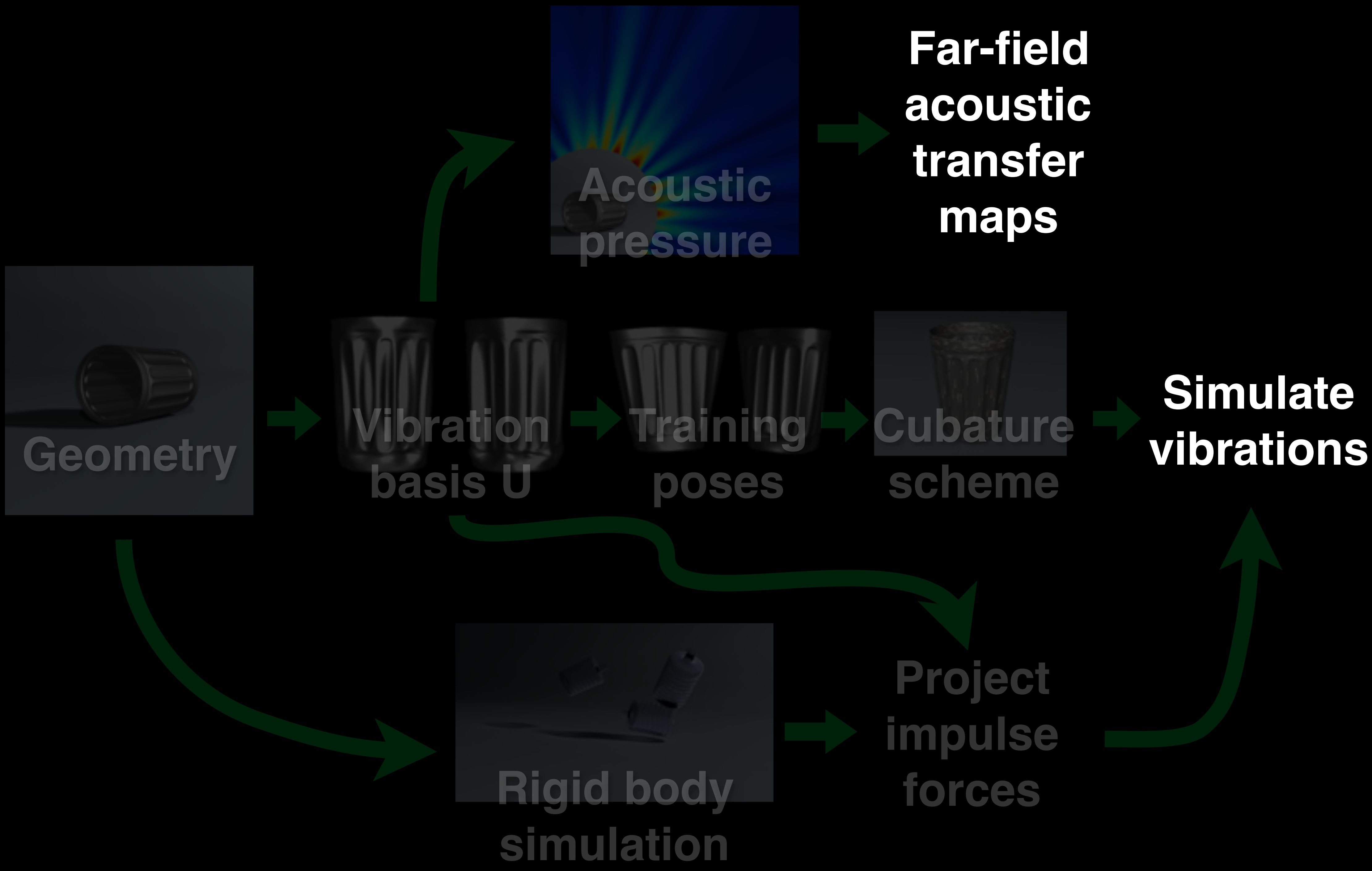

Sunday, December 13, 2009

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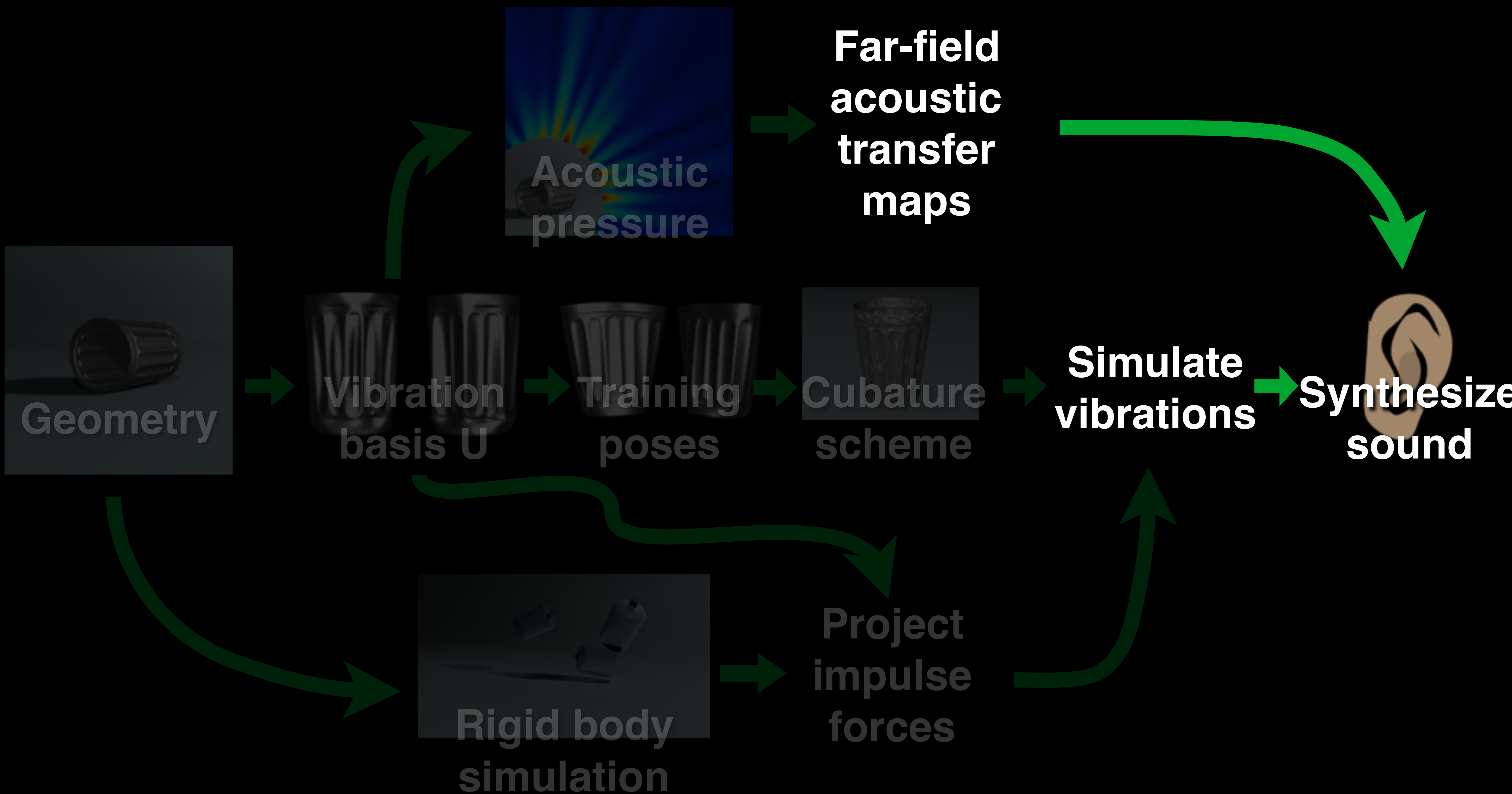
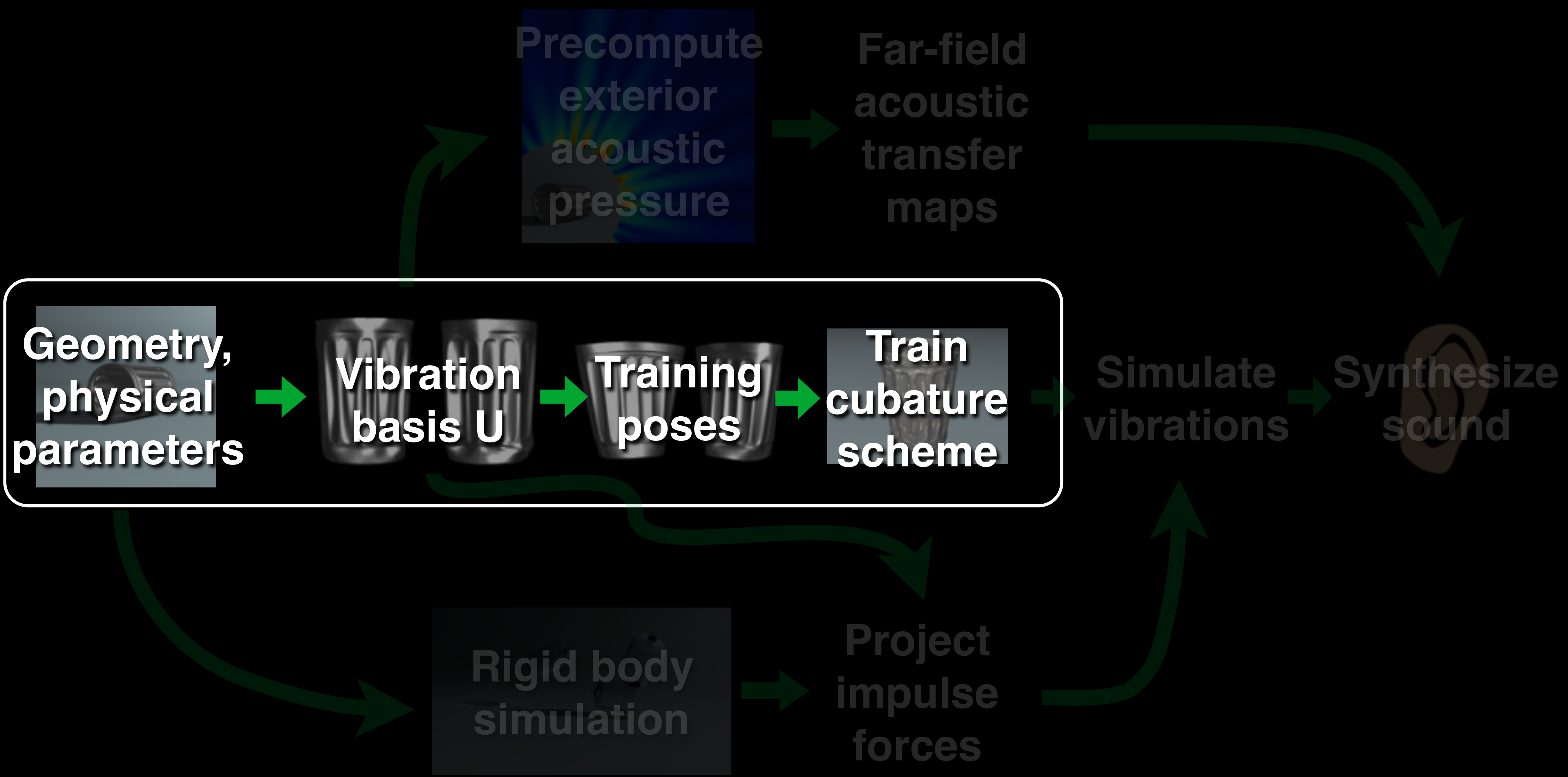


Model Reduction



Model Reduction
Related Work

Classical subspace integration, eg. [Bathe, 1996]

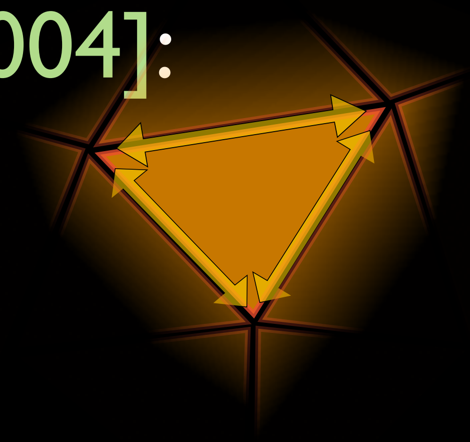
[Krysl et al. 2001] - Dimensional model reduction in non-linear finite element dynamics; “POD”/PCA

[Barbič et al. 2005] - Accelerated reduced force computation for St. Venant-Kirchhoff deformable models

[An et al. 2008] - Accelerated reduced force computation for general nonlinear materials

Model Reduction

Strain energy density (constant over triangle) [Gingold et al. 2004]:

$W(\mathbf{X}, \mathbf{x}) =$  $+$ 

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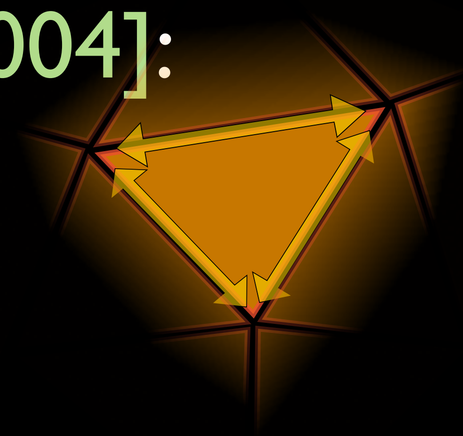
$W(\mathbf{X}, \mathbf{x}) =$  $+$ 

Strain Energy:

$$E(\mathbf{x}) = \int_S W(\mathbf{X}, \mathbf{x}) dS(\mathbf{X})$$

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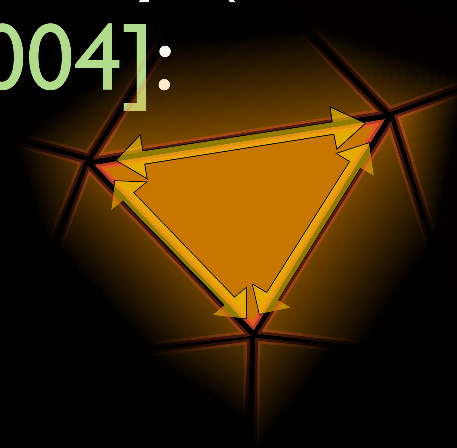
$W(\mathbf{X}, \mathbf{x}) =$  $+$ 

Strain Energy:

$$E(\mathbf{x}) = \int_S W(\mathbf{X}, \mathbf{x}) dS(\mathbf{X})$$

Internal forces:

$$\mathbf{f}(\mathbf{x}) = \nabla_x E(\mathbf{x}) = \int_S \nabla_x W(\mathbf{X}, \mathbf{x}) dS(\mathbf{X}) = \int_S G(\mathbf{X}, \mathbf{x}) dS(\mathbf{X})$$

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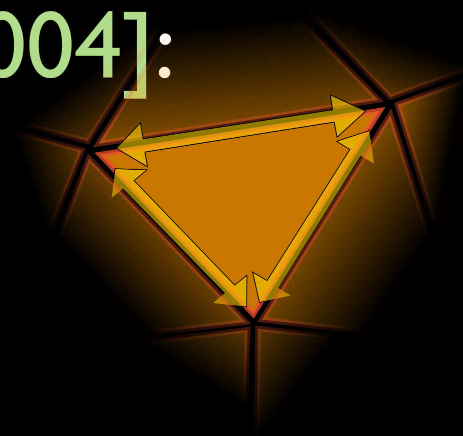

$$+$$

Strain Energy:

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Force density

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Nonlinear system of equations in displacements *u*

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{f}(\mathbf{u}) = \mathbf{f}_{external}$$

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Internal forces → $\mathbf{f}(\mathbf{u})$

Model Reduction

Nonlinear system of equations in displacements *u*

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Suppose some displacement basis given:

$$\mathbf{u} = \mathbf{U}\mathbf{q} \qquad \mathbf{U} \in \mathbb{R}^{3N \times r}$$

U = displacement basis

Model Reduction

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$\mathbf{u} = \mathbf{U}\mathbf{q}$ $\quad$ $\mathbf{U} \in \mathbb{R}^{3N \times r}$ $\quad$ *U* = displacement basis

$\mathbf{q} \in \mathbb{R}^{r}$ $\quad$ $r \ll 3N$ $\quad$ *q* = modal coordinates

3N ~ 100K $\quad$ *q* ~ hundreds

Model Reduction

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{f}(\mathbf{u}) = \mathbf{f}_{external} \qquad \mathbf{u} = \mathbf{U}\mathbf{q}$$

Eigen-modes and frequencies from linear modal analysis

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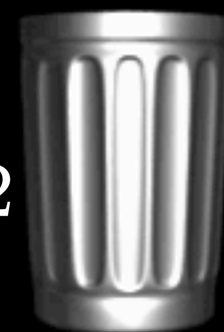
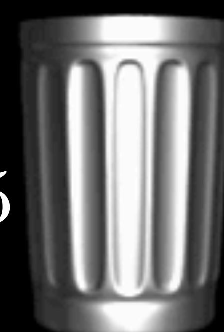
Eigen-modes and frequencies from linear modal analysis

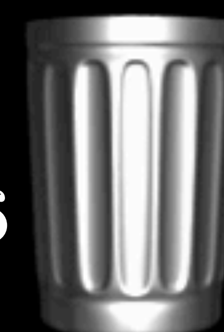
$U_{:,1}$ 

$U_{:,2}$ 

$U_{:,3}$ 

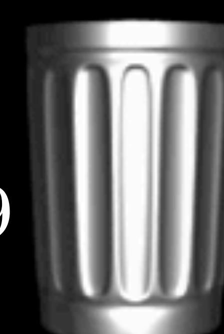
$U_{:,4}$ 

$U_{:,5}$ 

$U_{:,6}$ 

$U_{:,7}$ 

$U_{:,8}$ 

$U_{:,9}$ 

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Model Reduction

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Reduced internal forces

Model Reduction

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$$\tilde{\mathbf{M}}\ddot{\mathbf{q}} + \tilde{\mathbf{f}}(\mathbf{q}) = \tilde{\mathbf{f}}_{external}$$

Question: How to compute $\tilde{\mathbf{f}}(\mathbf{q})$?

Model Reduction

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Recall: Internal forces

Force density

$$\mathbf{f}(\mathbf{x}) = \int_S G(\mathbf{X}, \mathbf{x}) dS(\mathbf{X})$$

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Reduced force density

$$\tilde{\mathbf{f}}(\mathbf{q}) = \int_S \mathbf{U}^T G(\mathbf{X}, \mathbf{U}\mathbf{q}) dS(\mathbf{X}) = \int_S \mathbf{g}(\mathbf{X}, \mathbf{q}) dS(\mathbf{X})$$

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Problem: Matrix multiplies are *O(rN)*

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Problem: Matrix multiplies are *O(rN)*

Want: Reduced force evaluation independent of *N* (dependent only on *r*)

Model Reduction

$$\tilde{\mathbf{M}}\ddot{\mathbf{q}} + \tilde{\mathbf{f}}(\mathbf{q}) = \tilde{\mathbf{f}}_{external}$$

$$\tilde{\mathbf{f}}(\mathbf{q}) = \int_S \mathbf{U}^T G(\mathbf{X}, \mathbf{U}\mathbf{q}) dS(\mathbf{X}) = \int_S \mathbf{g}(\mathbf{X}, \mathbf{q}) dS(\mathbf{X})$$

Classical model reduction approach, eg. [Bathe 1996]

Individual explicit time steps more expensive (*O(rN)* instead of *O(N)*)

Has potential to significantly improve stability in explicit integration (larger time steps)

Optimized Cubature

Previous work

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- Input: Training poses and forces

$$\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_{N_T} \qquad \tilde{\mathbf{f}}(\mathbf{q_1}), \tilde{\mathbf{f}}(\mathbf{q_2}), \ldots, \tilde{\mathbf{f}}(\mathbf{q_{N_T}})$$

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- Introduced in [An et al. 2008]; tetrahedral models
- Approximate integral:

$$\tilde{\mathbf{f}}(\mathbf{q}) = \int_S \mathbf{g}(\mathbf{X}, \mathbf{q}) dS(\mathbf{X}) \approx \sum_{i=1}^{M} w_i \mathbf{g}(\mathbf{X}_i, \mathbf{q})$$

- Input: Training poses and forces

$$\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_{N_T} \qquad \tilde{\mathbf{f}}(\mathbf{q_1}), \tilde{\mathbf{f}}(\mathbf{q_2}), \ldots, \tilde{\mathbf{f}}(\mathbf{q_{N_T}})$$

- Output: points $\mathbf{X}_i$ and optimized weights w_i

Optimized Cubature

Previous work

$$\tilde{\mathbf{f}}(\mathbf{q}) = \int_S \mathbf{g}(\mathbf{X}, \mathbf{q}) dS(\mathbf{X}) \approx \sum_{i=1}^{M} w_i \mathbf{g}(\mathbf{X}_i, \mathbf{q})$$

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Result: $O(r^2)$ approximation of $\tilde{\mathbf{f}}(\mathbf{q})$

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Result: $O(r^2)$ approximation of $\tilde{\mathbf{f}}(\mathbf{q})$

$O(r^2)$ explicit time steps for system - reduced from $O(rN)$

$$\tilde{\mathbf{M}}\ddot{\mathbf{q}} + \tilde{\mathbf{f}}(\mathbf{q}) = \tilde{\mathbf{f}}_{external}$$

Optimized Cubature

Applying Cubature to Thin Shells

$$\tilde{\mathbf{f}}(\mathbf{q}) = \int_S \mathbf{g}(\mathbf{X}, \mathbf{q}) dS(\mathbf{X})$$

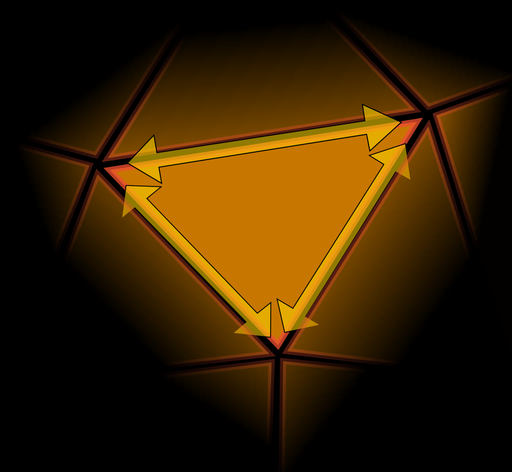
Optimized Cubature

Applying Cubature to Thin Shells

$$\tilde{\mathbf{f}}(\mathbf{q}) = \int_S \mathbf{g}(\mathbf{X}, \mathbf{q}) dS(\mathbf{X})$$

Strain energy density: constant over each triangle
(same is true for reduced force density)

$W(\mathbf{X}, \mathbf{x}) =$



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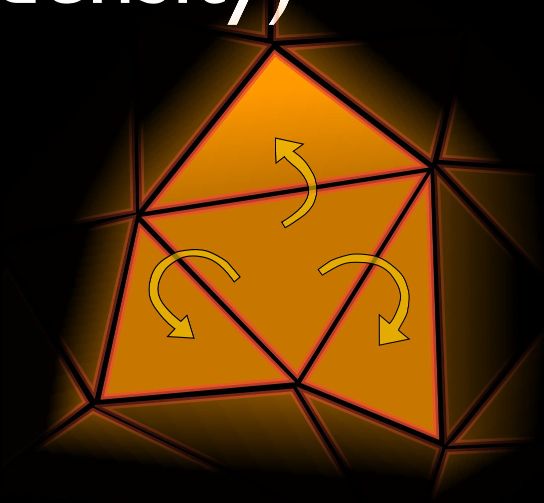
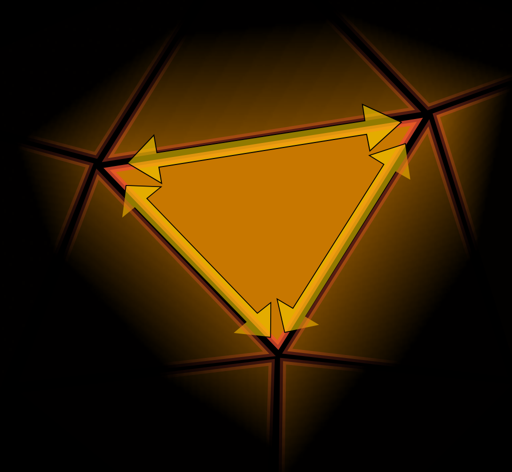
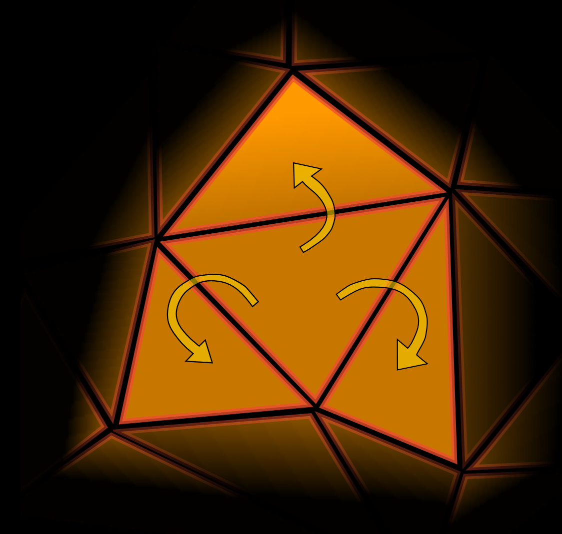


Optimized Cubature

Applying Cubature to Thin Shells

Internal forces: sum over triangles

$$\tilde{\mathbf{f}}(\mathbf{q}) = \int_S \mathbf{g}(\mathbf{X}, \mathbf{q}) dS(\mathbf{X}) = \sum_{i=1}^{N_T} A_i \mathbf{g}(\mathbf{X}_{T_i}, \mathbf{q})$$

$\mathbf{g}(\mathbf{X}_{T_i}, \mathbf{q}) =$  $+$ 

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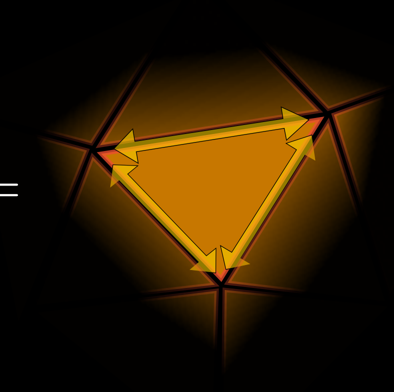
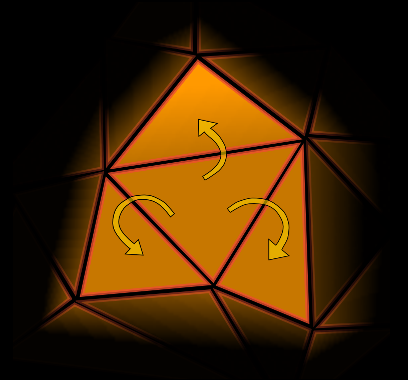
$\mathbf{g}(\mathbf{X}_{T_i}, \mathbf{q}) =$  $+$ 

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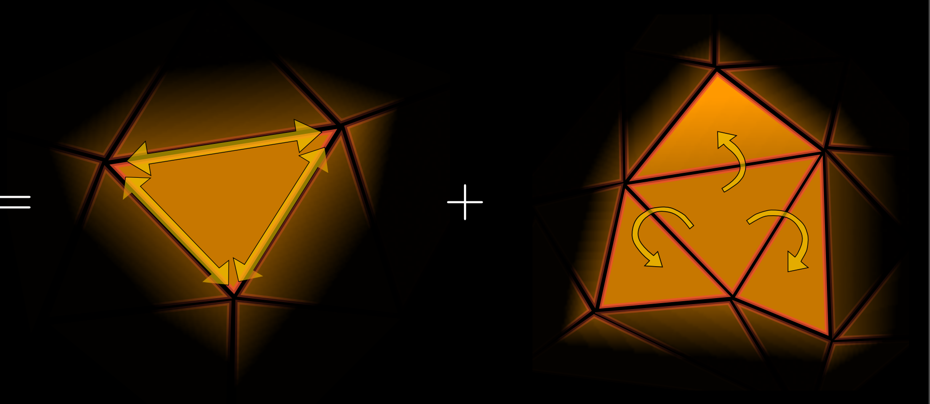
$$\mathbf{g}(\mathbf{X}_{T_i}, \mathbf{q}) = \quad [\text{stretching}] \quad + \quad [\text{bending}]$$



Optimized Cubature

Applying Cubature to Thin Shells

Internal forces: sum over triangles

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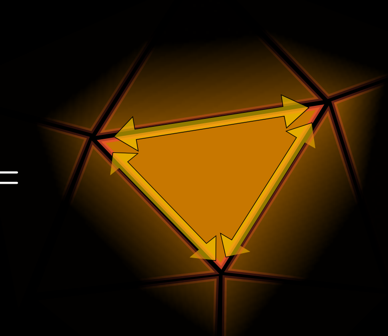
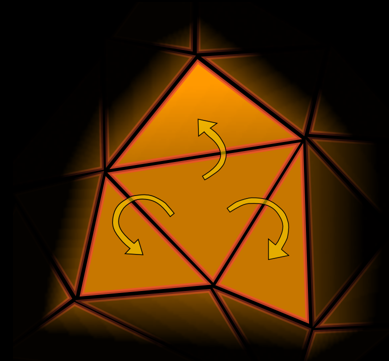
Choose subset and weights: $\mathbf{g}(\mathbf{X}_{T_i}, \mathbf{q}) =$  $+$ 

$$\left.\begin{array}{c} \{t_1, \ldots, t_C\} \subset \{T_1, \ldots, T_{N_T}\} \\ \{w_1, \ldots, w_C\} \end{array}\right\} \quad C \ll N_T$$

Optimized Cubature

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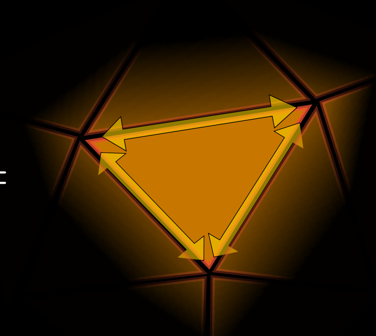
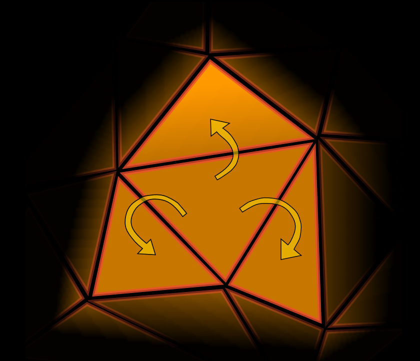
Choose subset and weights: $\mathrm{g}(\mathrm{X}_{T_i}, \mathrm{q}) =$  $+$ 

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$$\sum_{i=1}^{N_T} A_i \mathbf{g}(\mathbf{X}_{T_i}, \mathbf{q}) \approx \sum_{i=1}^{C} w_i A_i \mathbf{g}(\mathbf{X}_{t_i}, \mathbf{q})$$

Optimized Cubature

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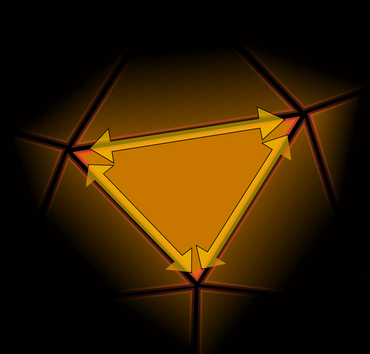
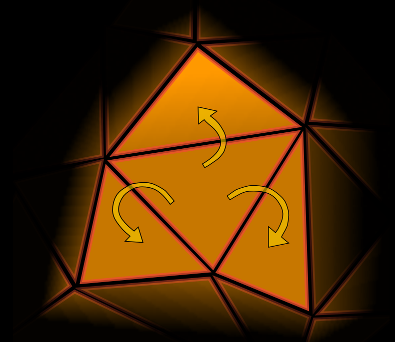
Choose subset and weights: $\mathbf{g}(\mathbf{X}_{T_i}, \mathbf{q}) =$  $+$ 

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Use cubature training to choose subset/weights

Optimized Cubature

800 element cubature scheme (78K triangles)

Model Reduction

Summary

Model Reduction

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- What we keep from linear modal sound synthesis:

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  - Small displacement assumption

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$$\mathbf{u} = \mathbf{U}\mathbf{q}$$

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Dimensional model reduction:
Significantly increases stable time step size

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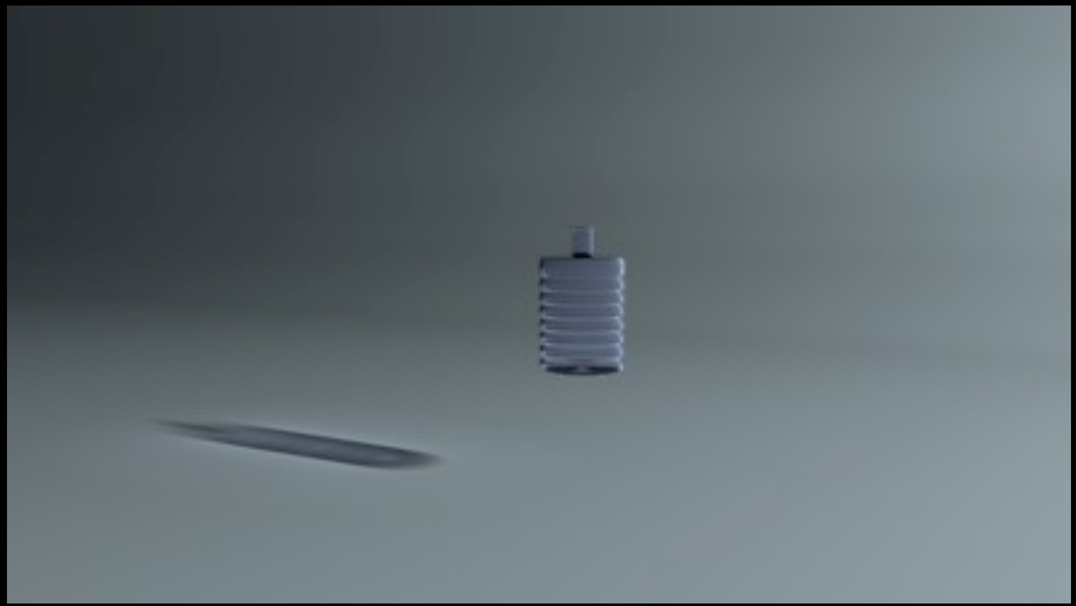

Full simulation: ~11M time steps per second

Reduced simulation: 44100 time steps per second

Model Reduction

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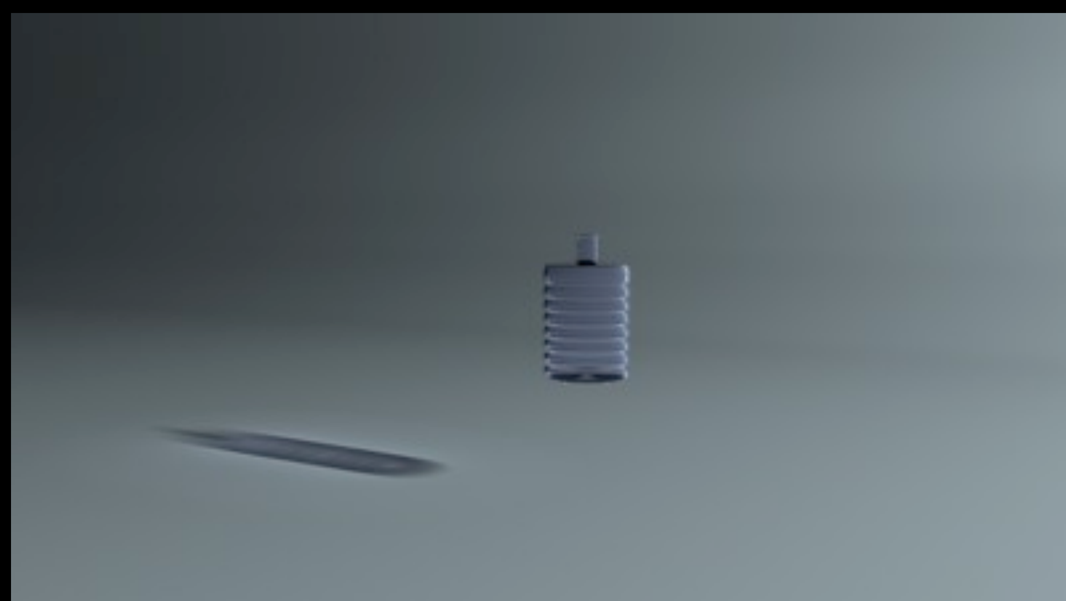

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19 days vs. 15 hours for 5s of audio

Model Reduction

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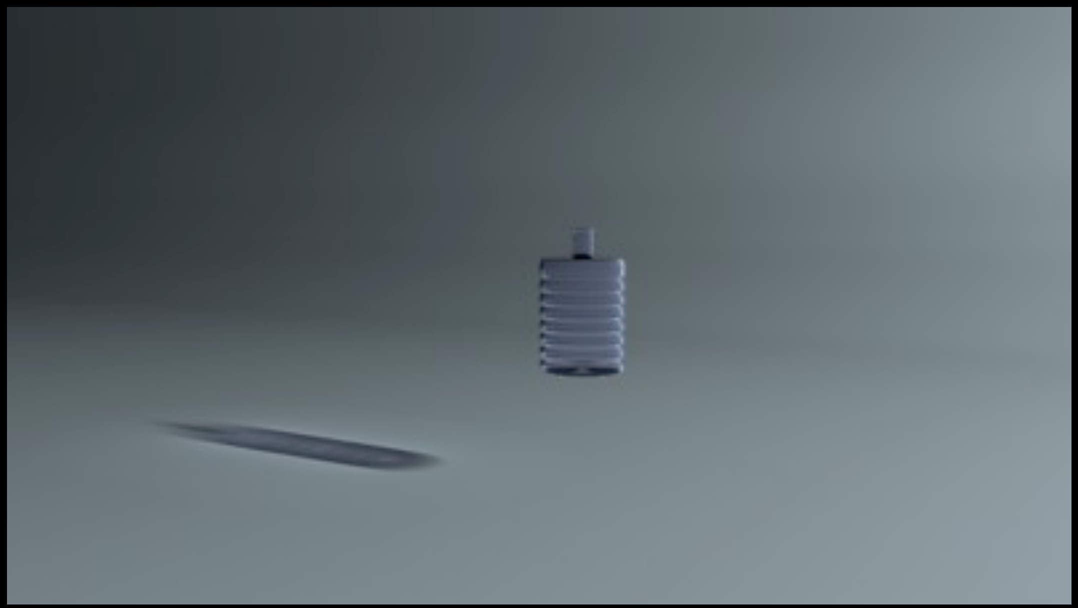

Model Reduction

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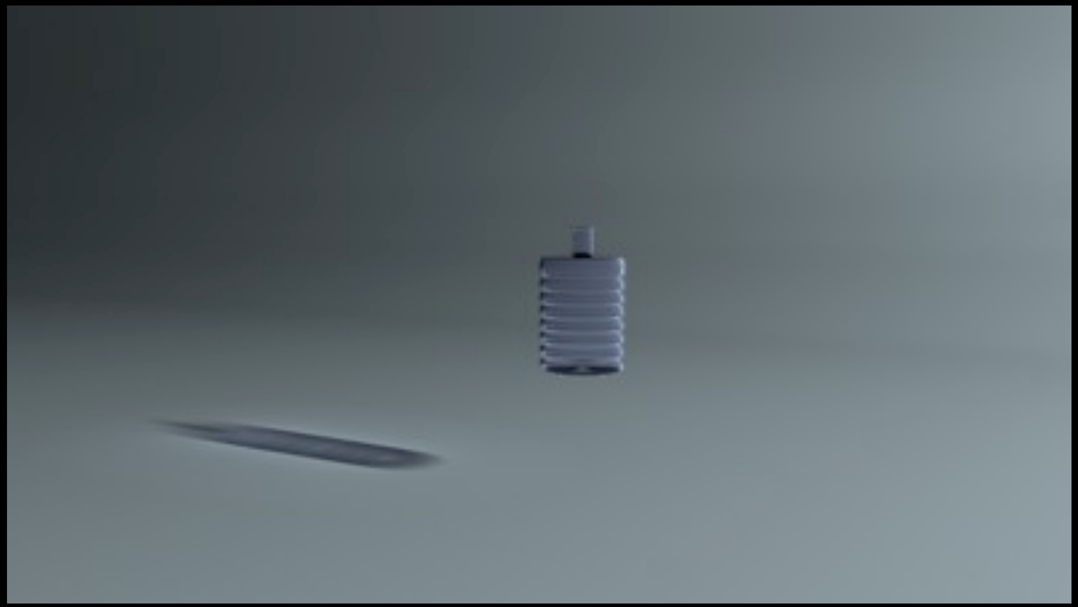
Cubature algorithm:
Reduces time step cost from $O(rN)$ to $O(r^2)$



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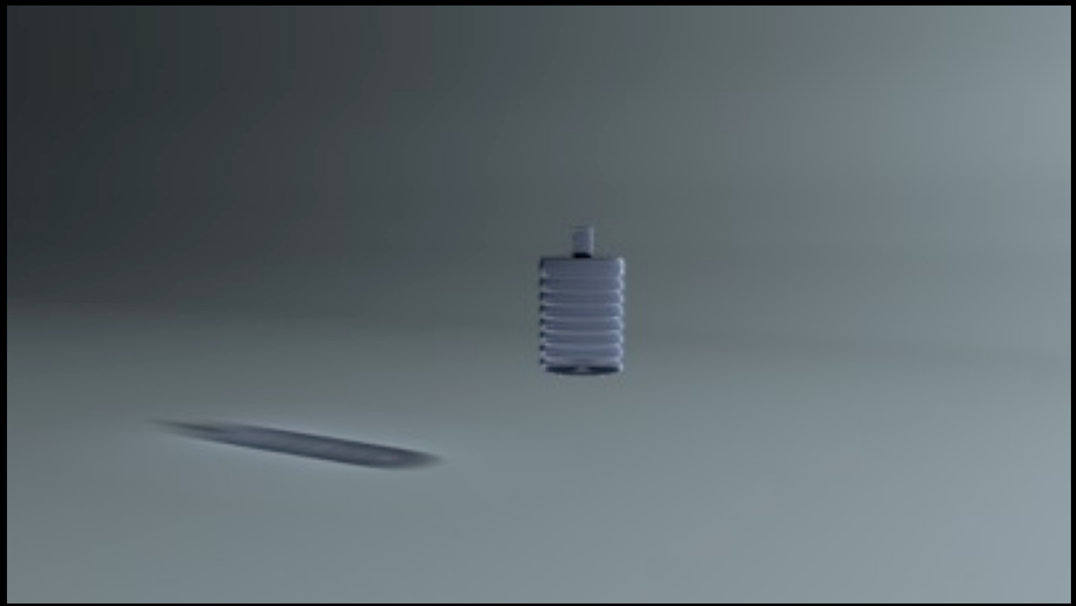

15 hours vs. 1.5 hours for 5s of audio

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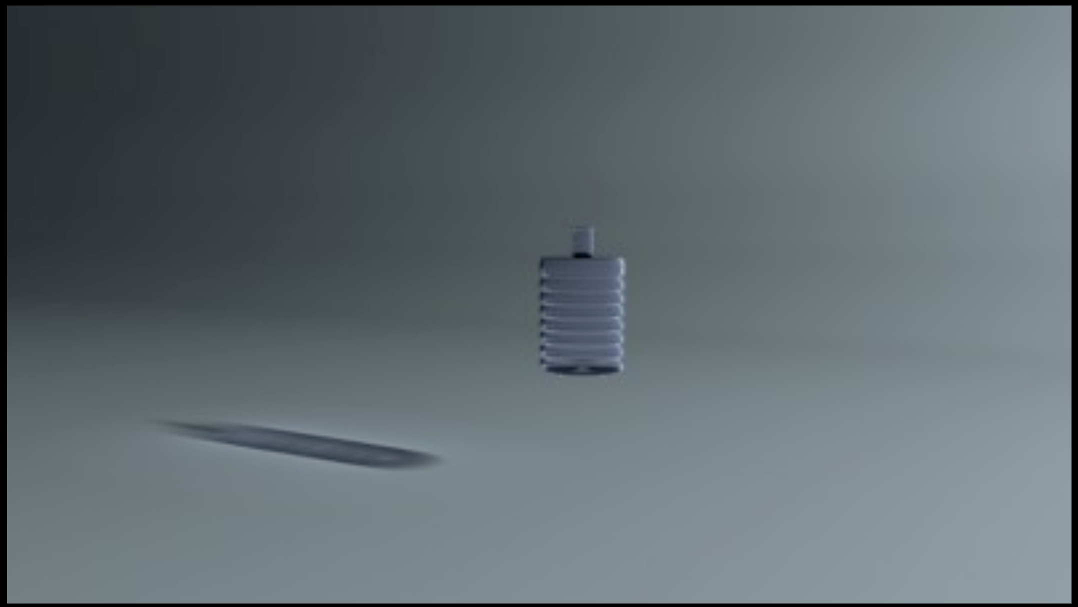
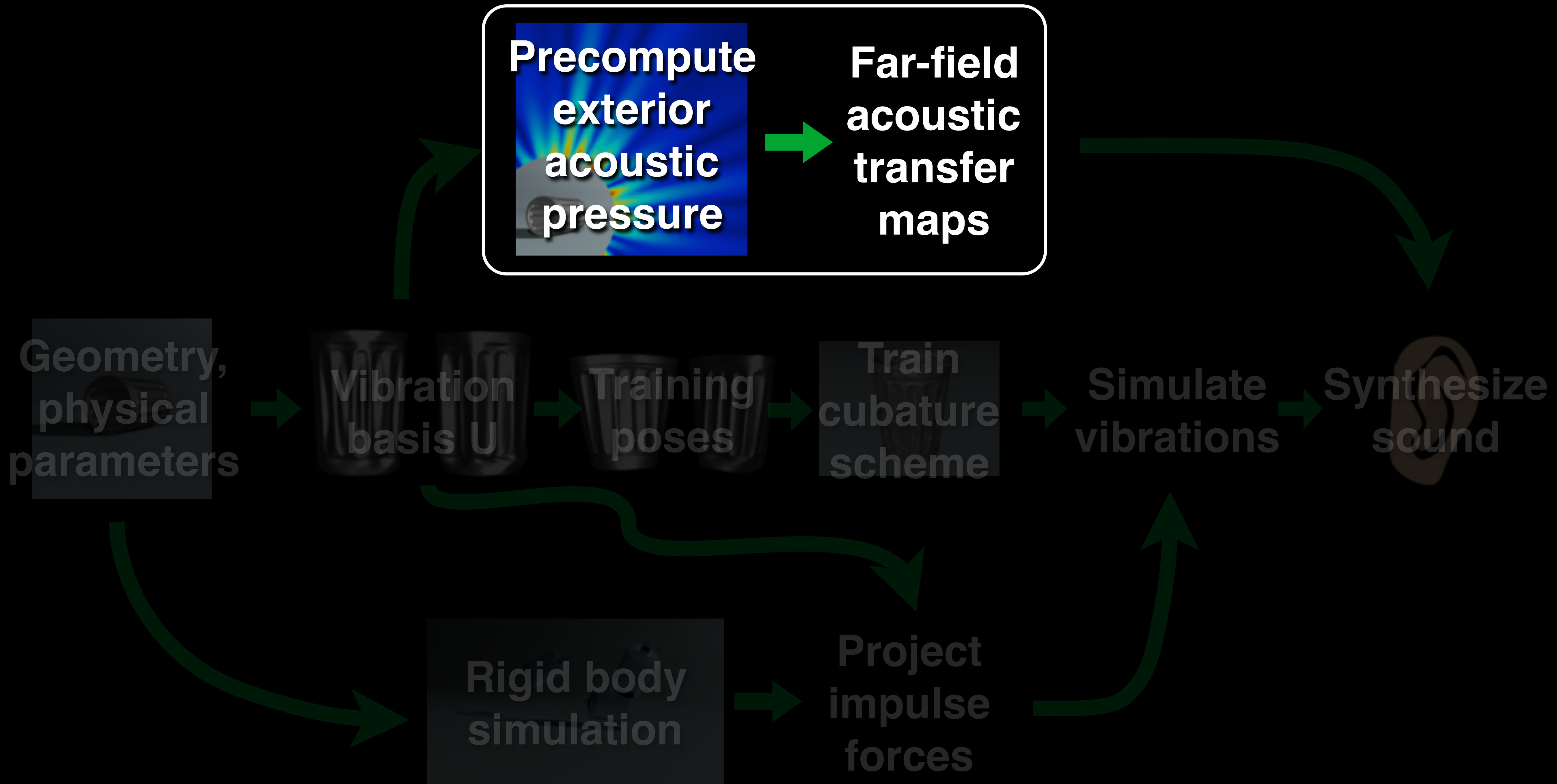


15 hours vs. 1.5 hours for 5s of audio

Overall: Larger, cheaper time steps

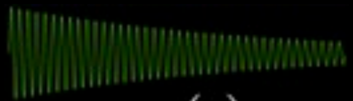
Approximating Acoustic Transfer



Approximating Acoustic Transfer

Approximating Acoustic Transfer

Sum of modal amplitudes:



$$p(\mathbf{x}, t) = \frac{q_1(t) + q_2(t) + \cdots}{r}$$

Approximating Acoustic Transfer

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Or, weighted sum:

$$p(\mathbf{x}, t) = \sum_{i=1}^{N_{modes}} q_i(t)\, |p_i(\mathbf{x})|$$

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"Acoustic transfer function" (far-field, low frequency, monopole approximation)

Approximating Acoustic Transfer

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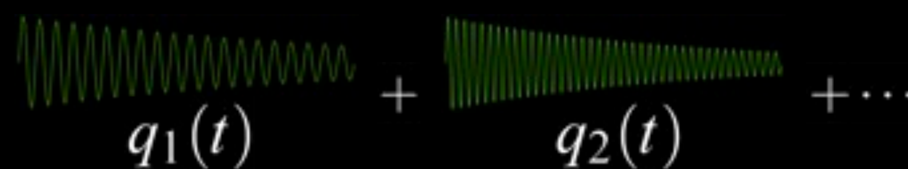

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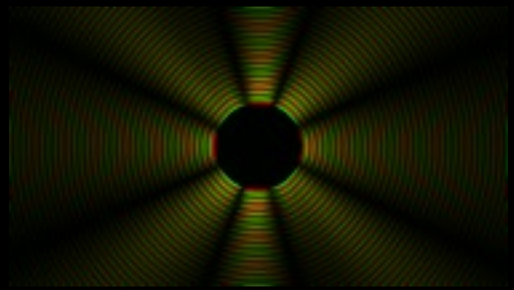
"Acoustic transfer function" (far-field, low frequency, monopole approximation)

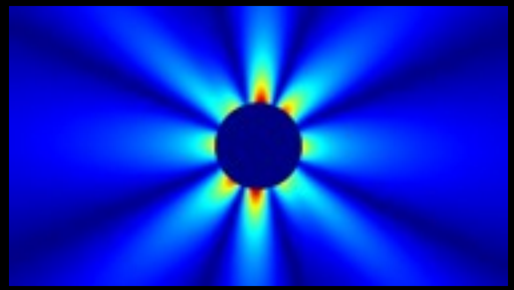
In general: $\left(\nabla^2 + k_i^2\right) p_i(\mathbf{x}) = 0$

Approximating Acoustic Transfer

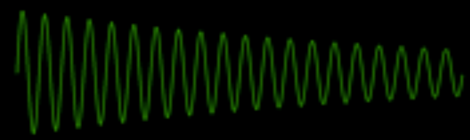
Acoustic Transfer function: $p(\mathbf{x})$



Amplitude of unit vibration: $|p(\mathbf{x})|$



Modal sound contribution: $|p(\mathbf{x})|q(t)$



Problem: Must evaluate $p(\mathbf{x})$ for each time sample, mode and object

Standard solution techniques (eg. BEM) too expensive

Approximating Acoustic Transfer

Approximating Acoustic Transfer

- “Precomputed Acoustic Transfer” [James et al. 2006]
    - Approximate $p_i(\mathbf{x})$ with sum of simple source functions

Approximating Acoustic Transfer

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  - Approximate $p_i(\mathbf{x})$ with sum of simple source functions
- Problems with this approach:
  - Difficult fitting problem for high frequencies
  - Increasingly costly transfer evaluations with higher frequencies (more sources needed)

Approximating Acoustic Transfer

Exploiting radial structure

Approximating Acoustic Transfer

Exploiting radial structure

- Ignore behavior near to the object (eg. within 2-3 bounding sphere radii)

Approximating Acoustic Transfer

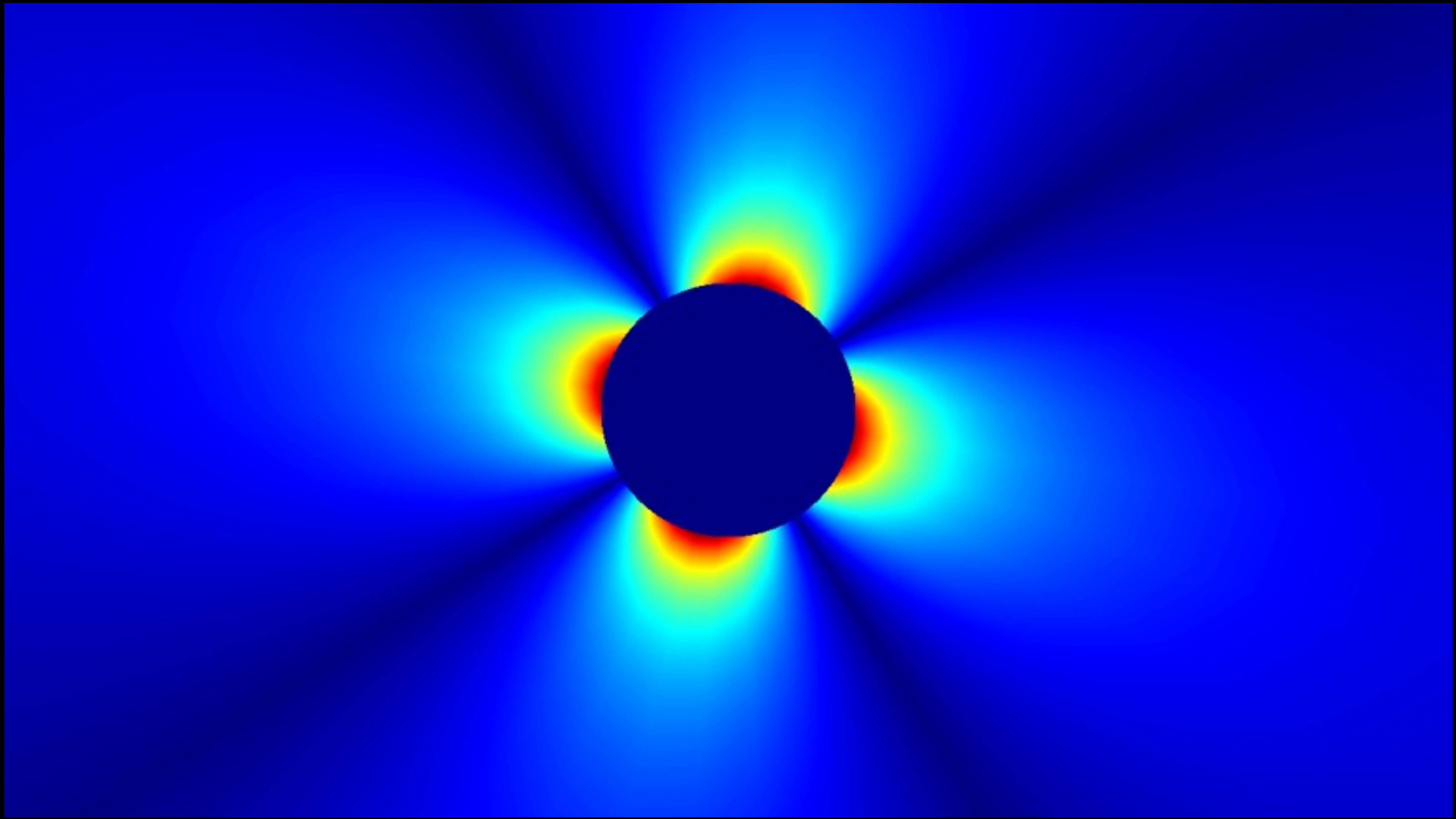
Exploiting radial structure

Ignore behavior near to the object (eg. within 2-3 bounding sphere radii)

Look for structure in far field pressure behavior

Approximating Acoustic Transfer

Exploiting radial structure



Approximating Acoustic Transfer

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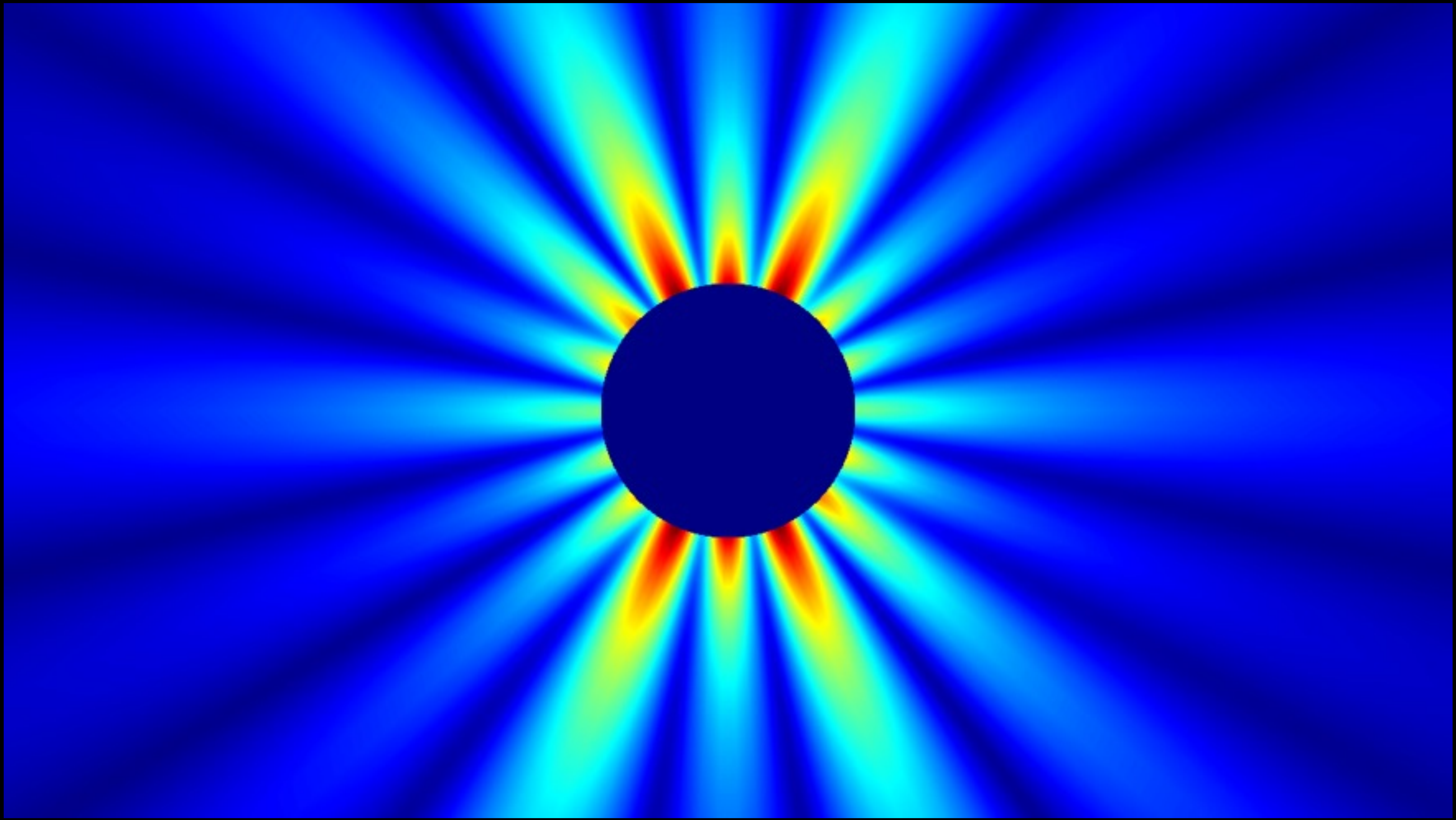

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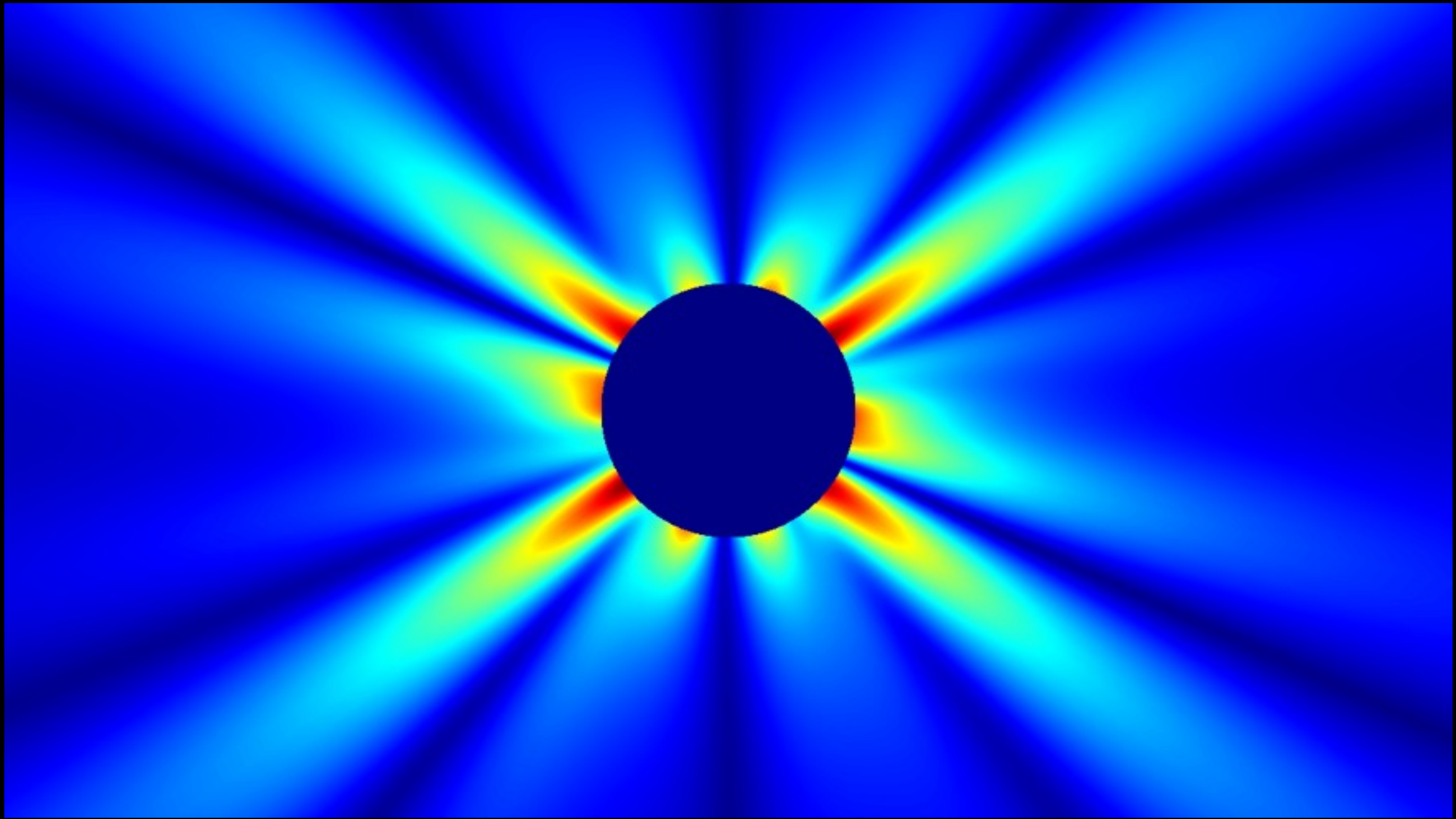

Approximating Acoustic Transfer

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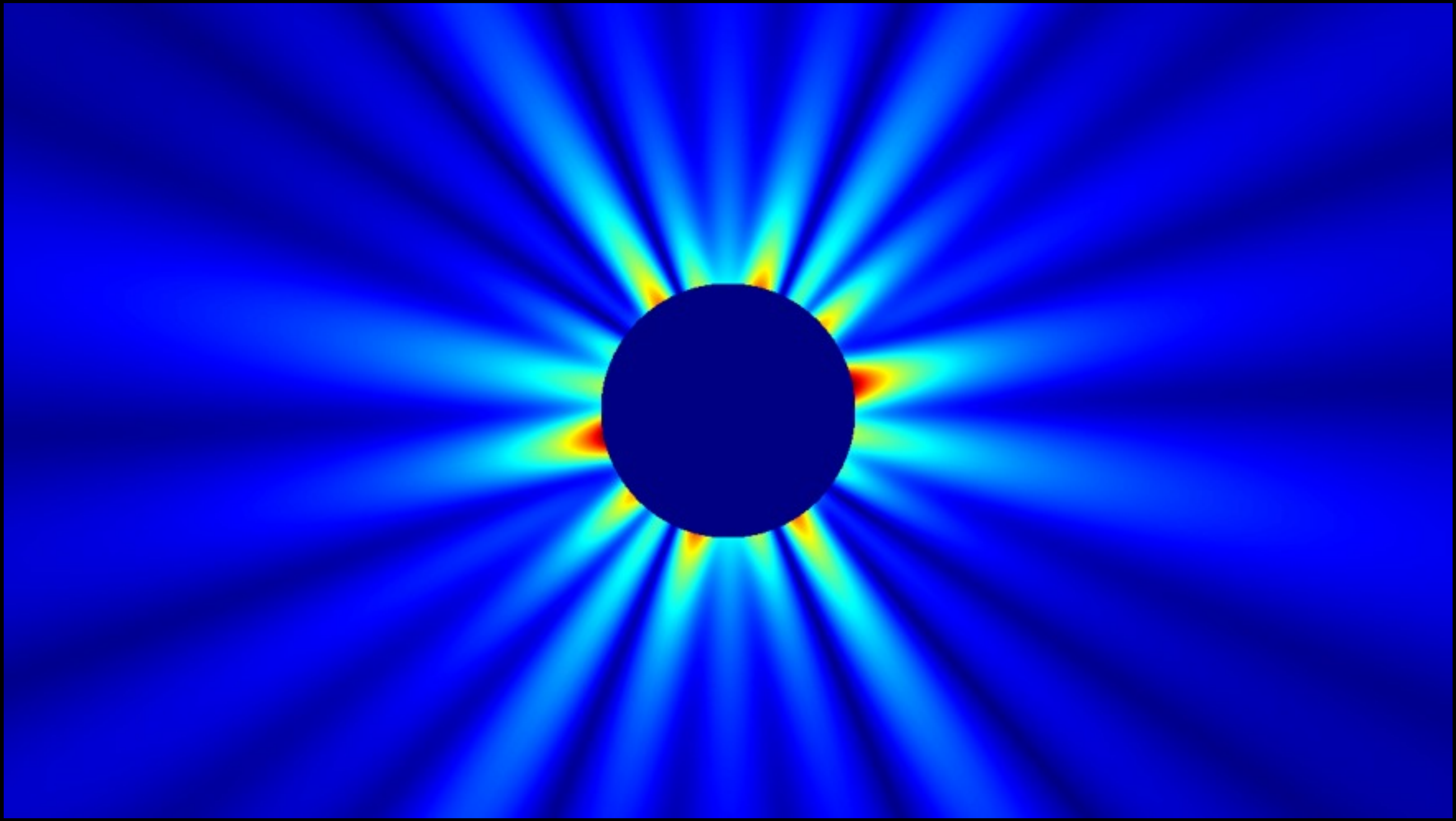

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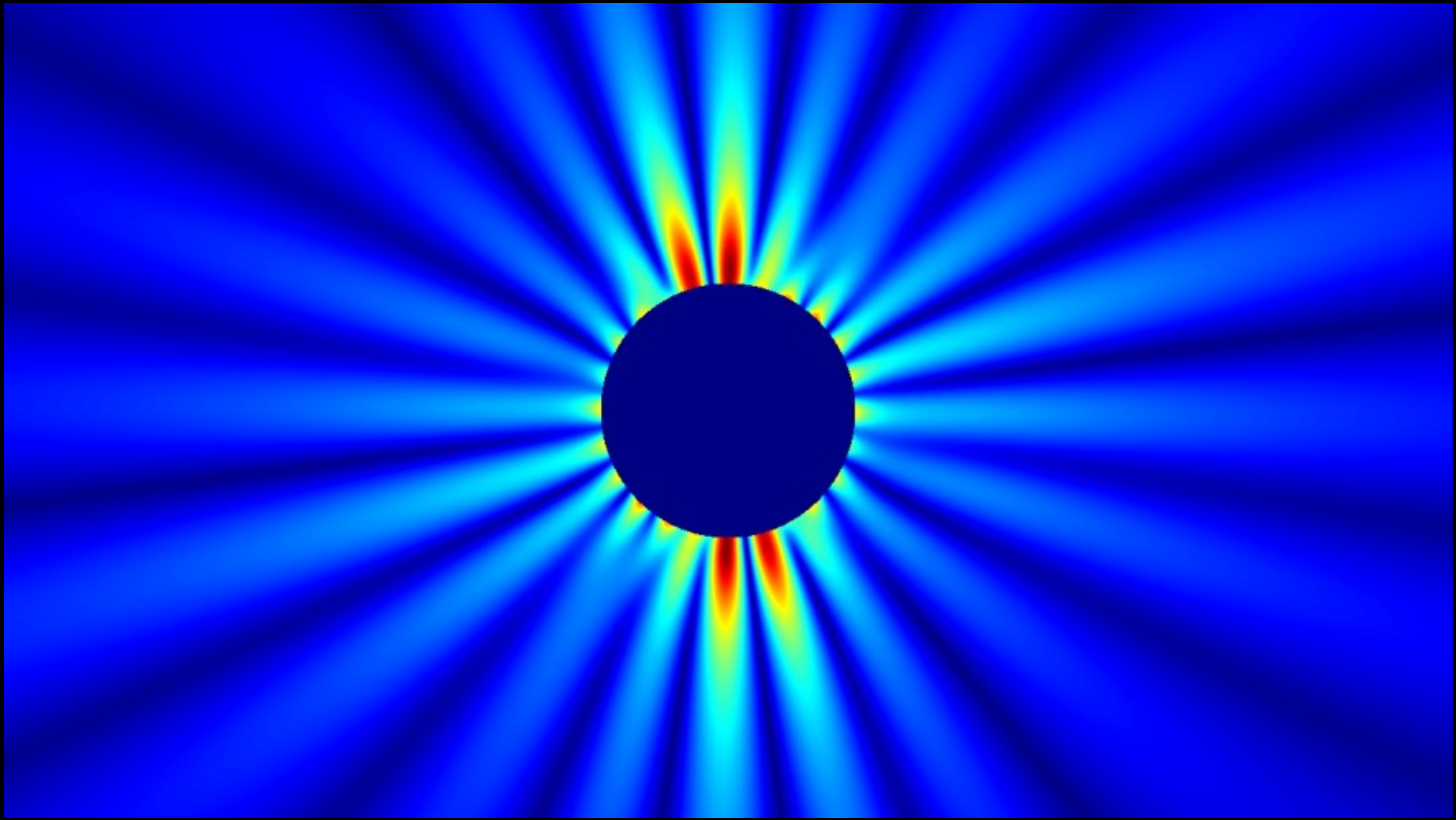

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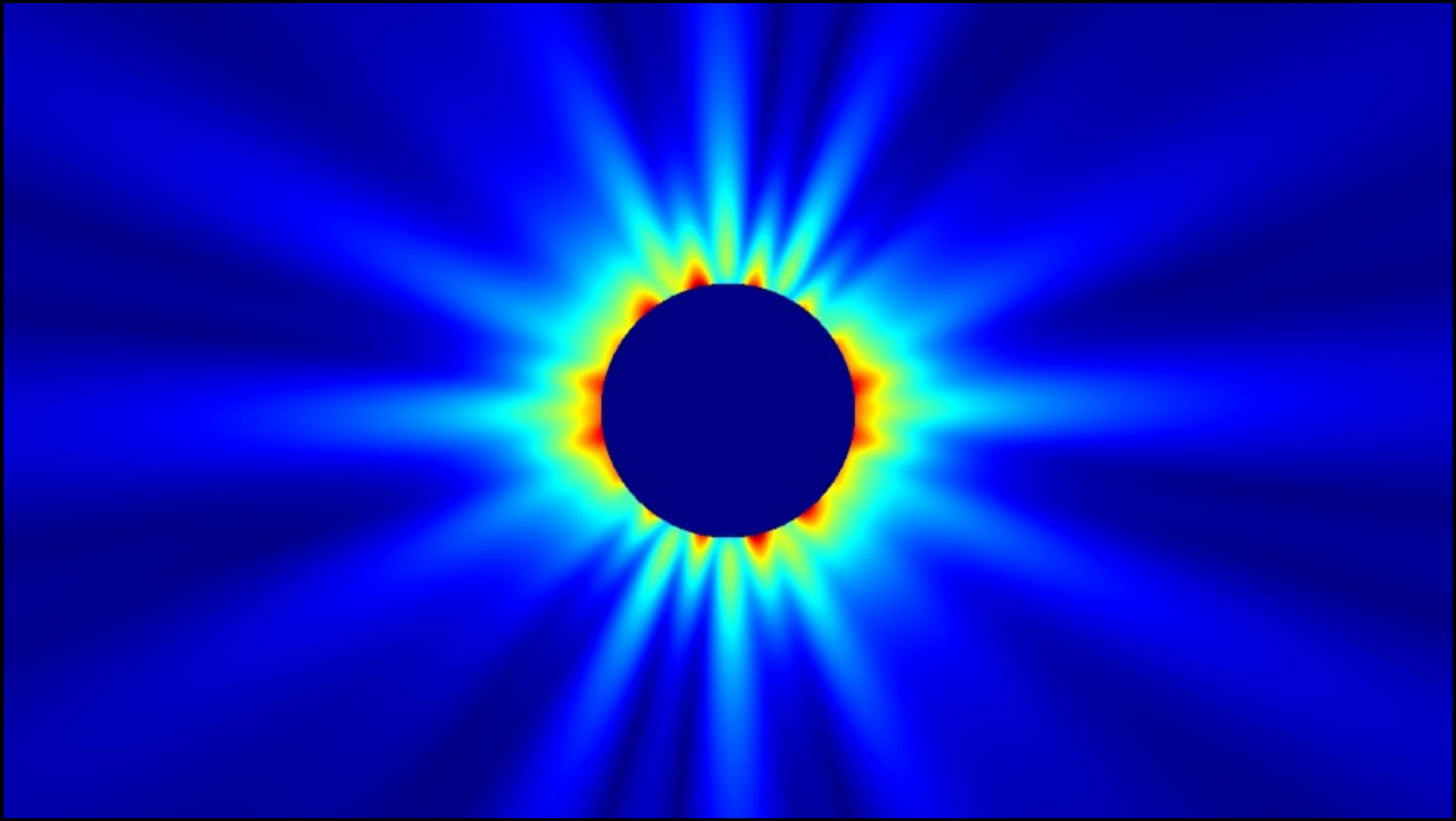

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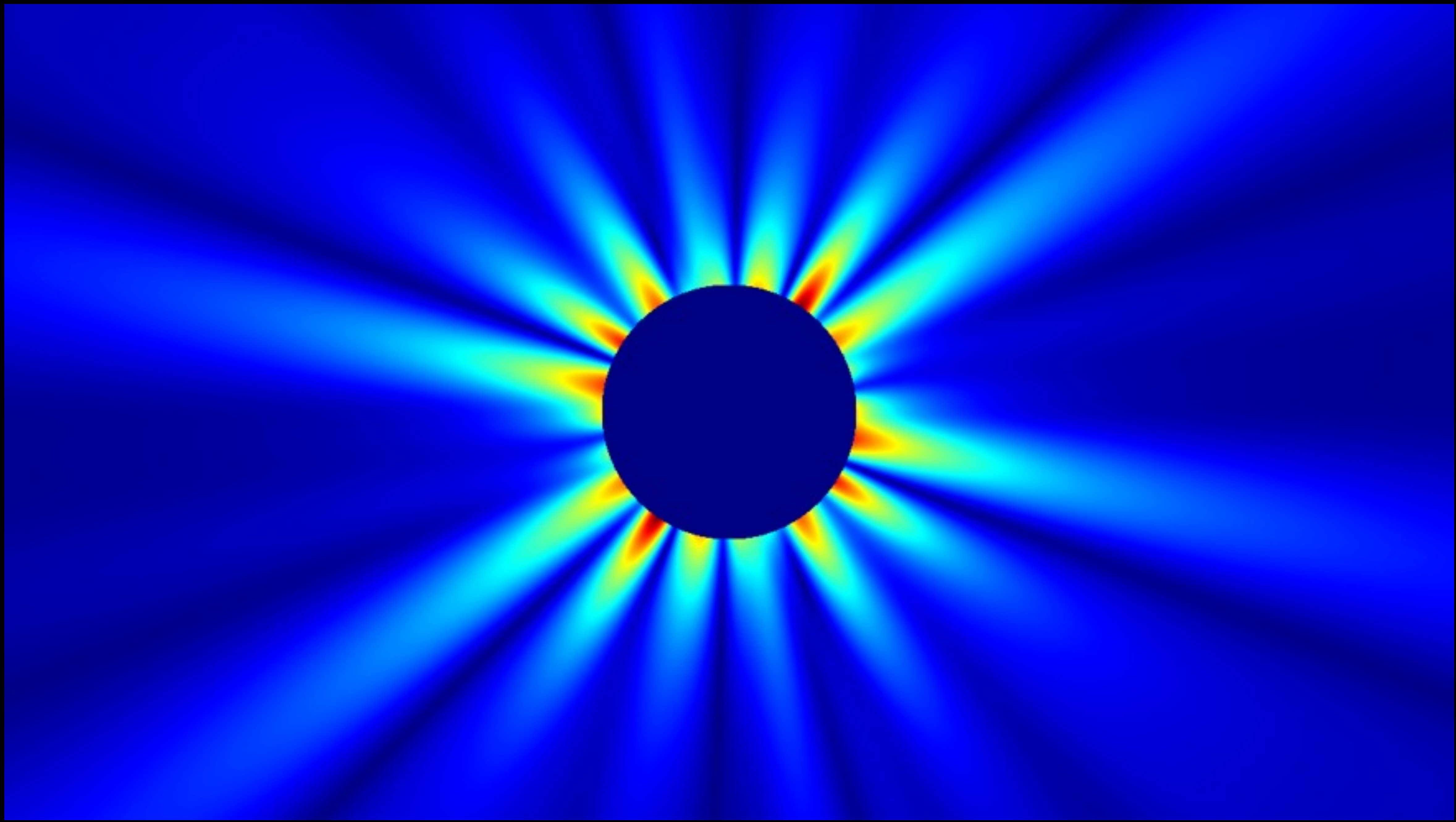

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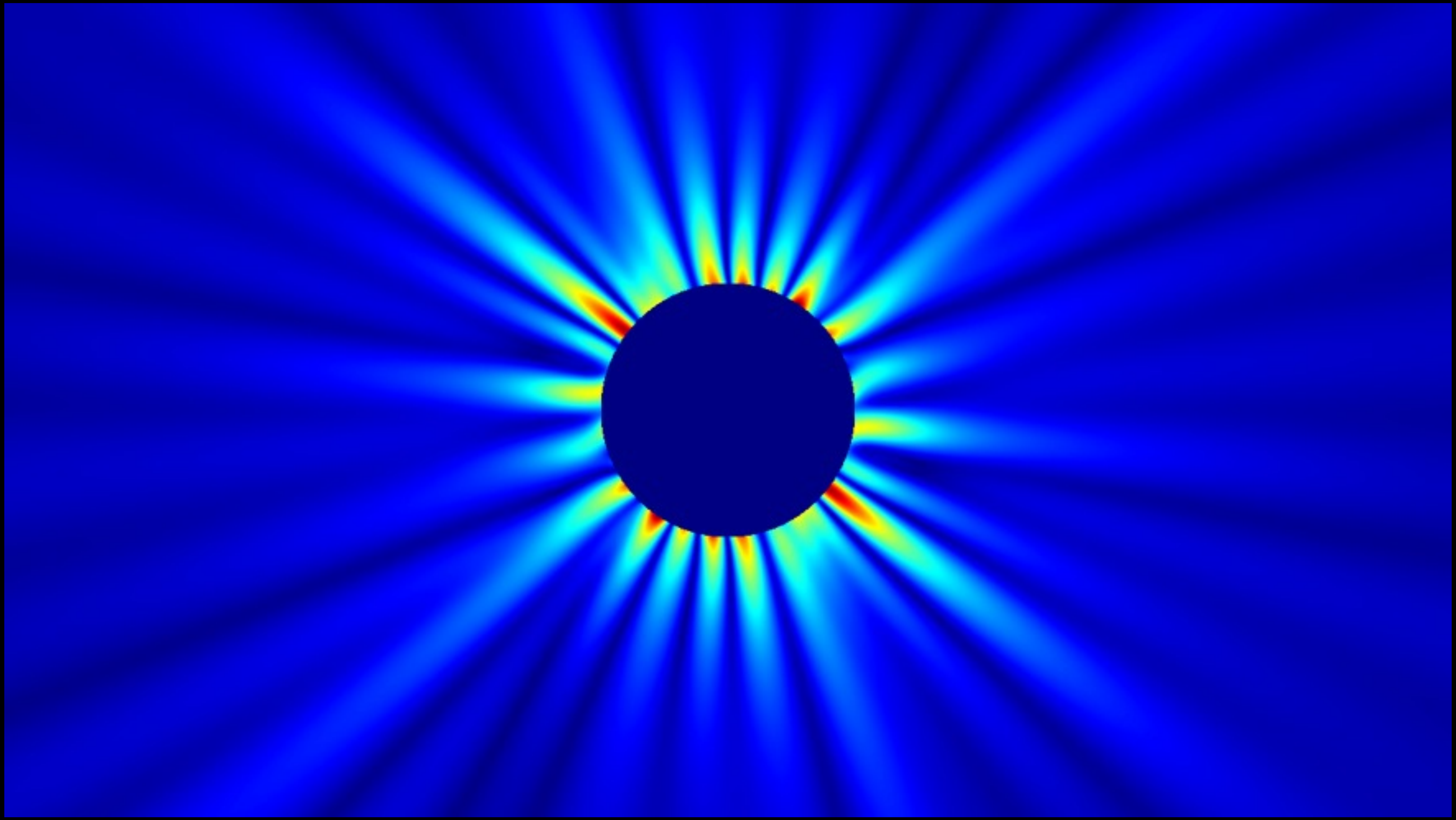

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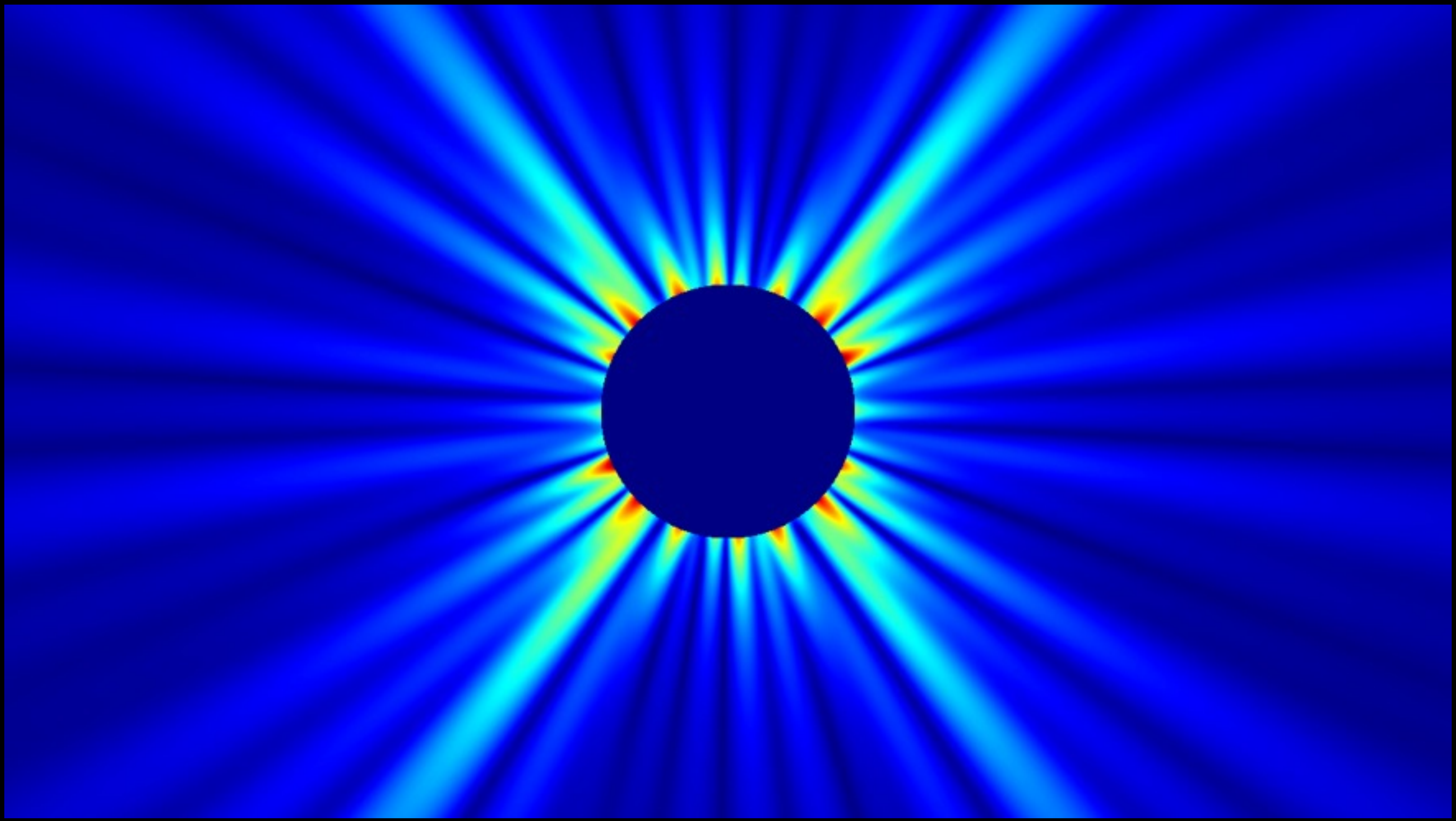

Approximating Acoustic Transfer

Suppose the pressure field surrounding an object is known:

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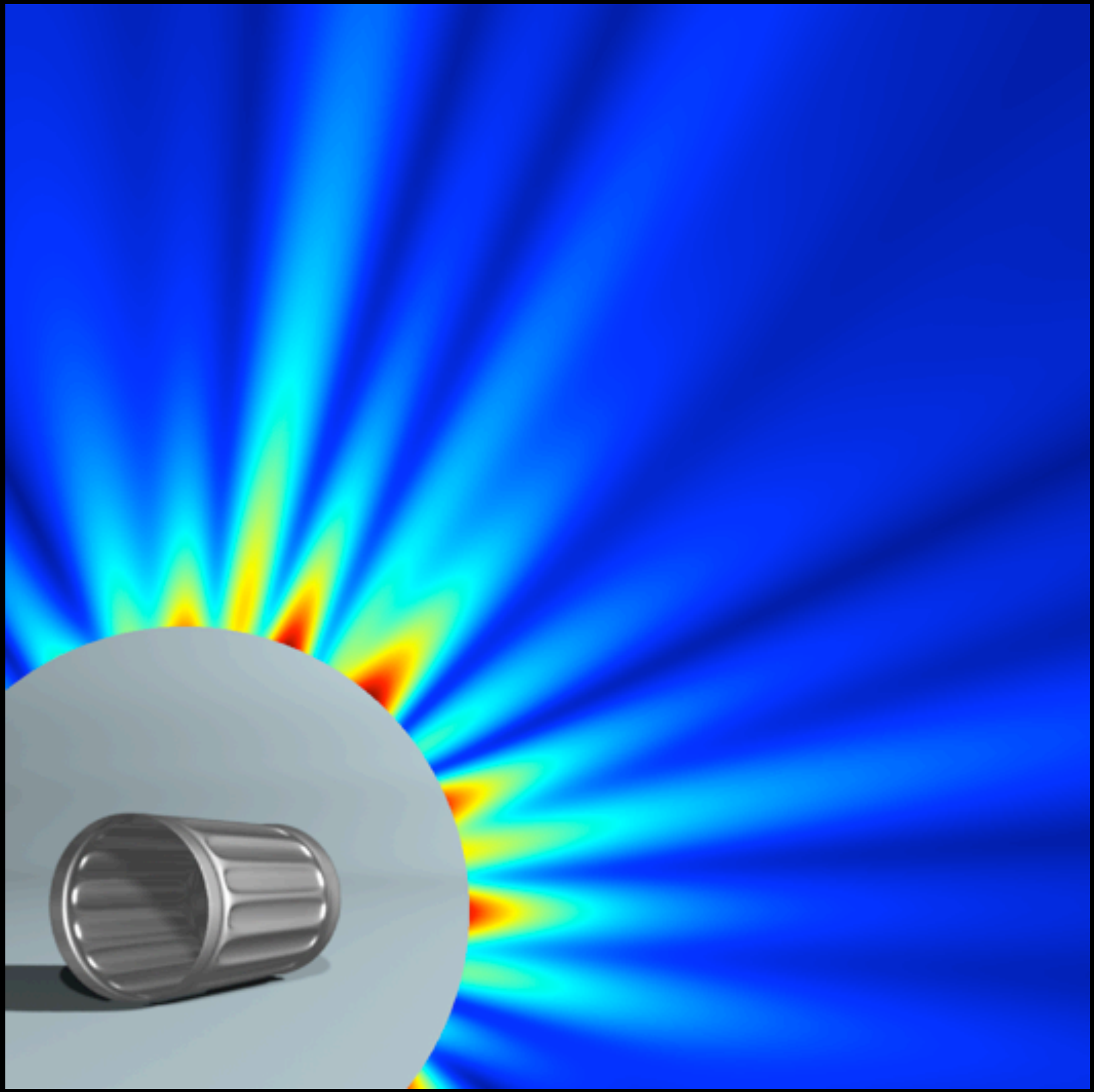
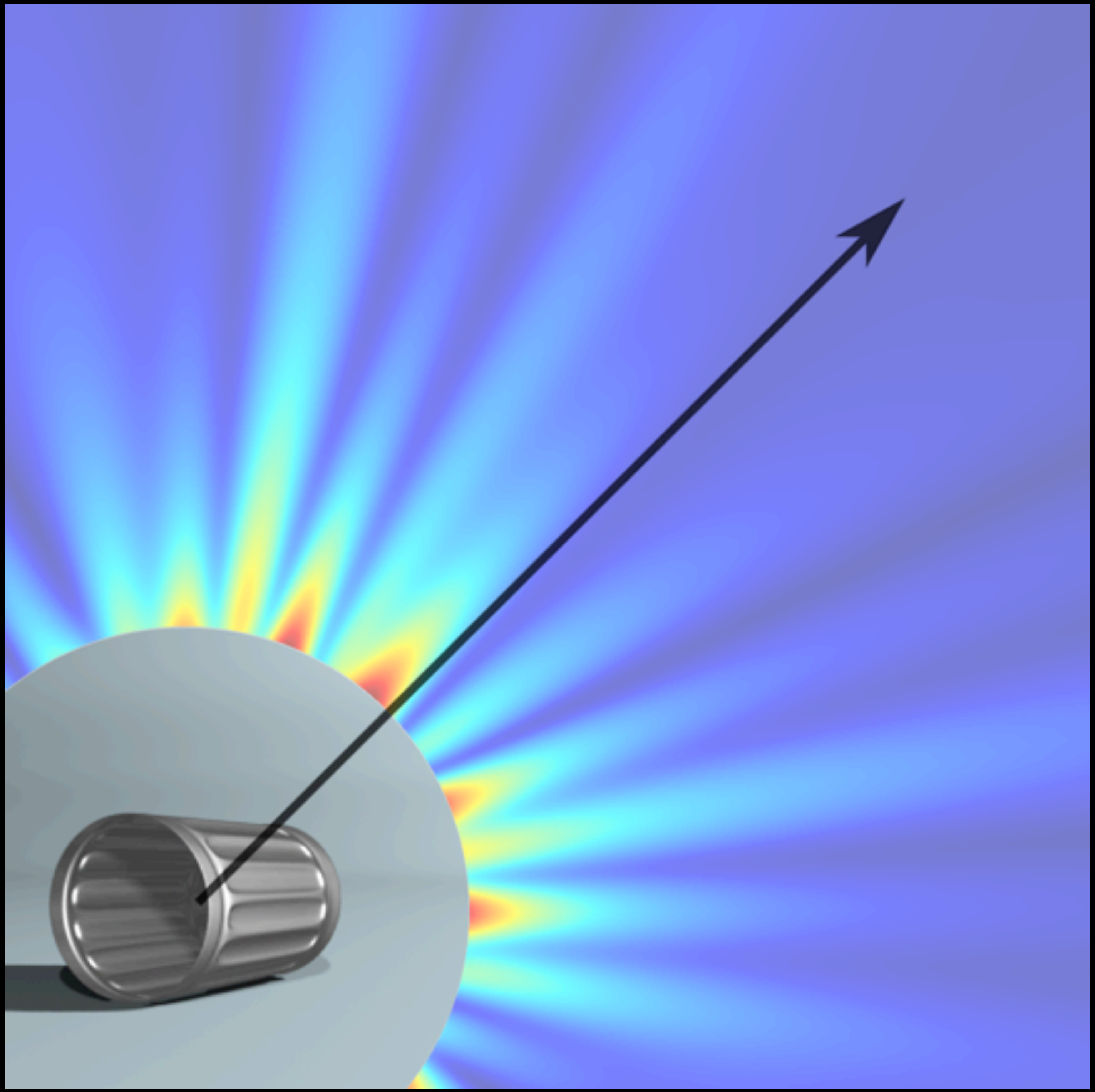


Approximating Acoustic Transfer

Fix radial direction:

Pre-compute estimate in this direction



Approximating Acoustic Transfer

Consider an M-term asymptotic expansion

$$p(x) \sim h_0(kR) \left\{ \Psi_1(\theta,\phi) + \frac{\Psi_2(\theta,\phi)}{kR} + \cdots + \frac{\Psi_M(\theta,\phi)}{(kR)^{M-1}} \right\}$$

$$-\frac{ie^{-ikR}}{kR}$$

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Unknowns

Approximating Acoustic Transfer

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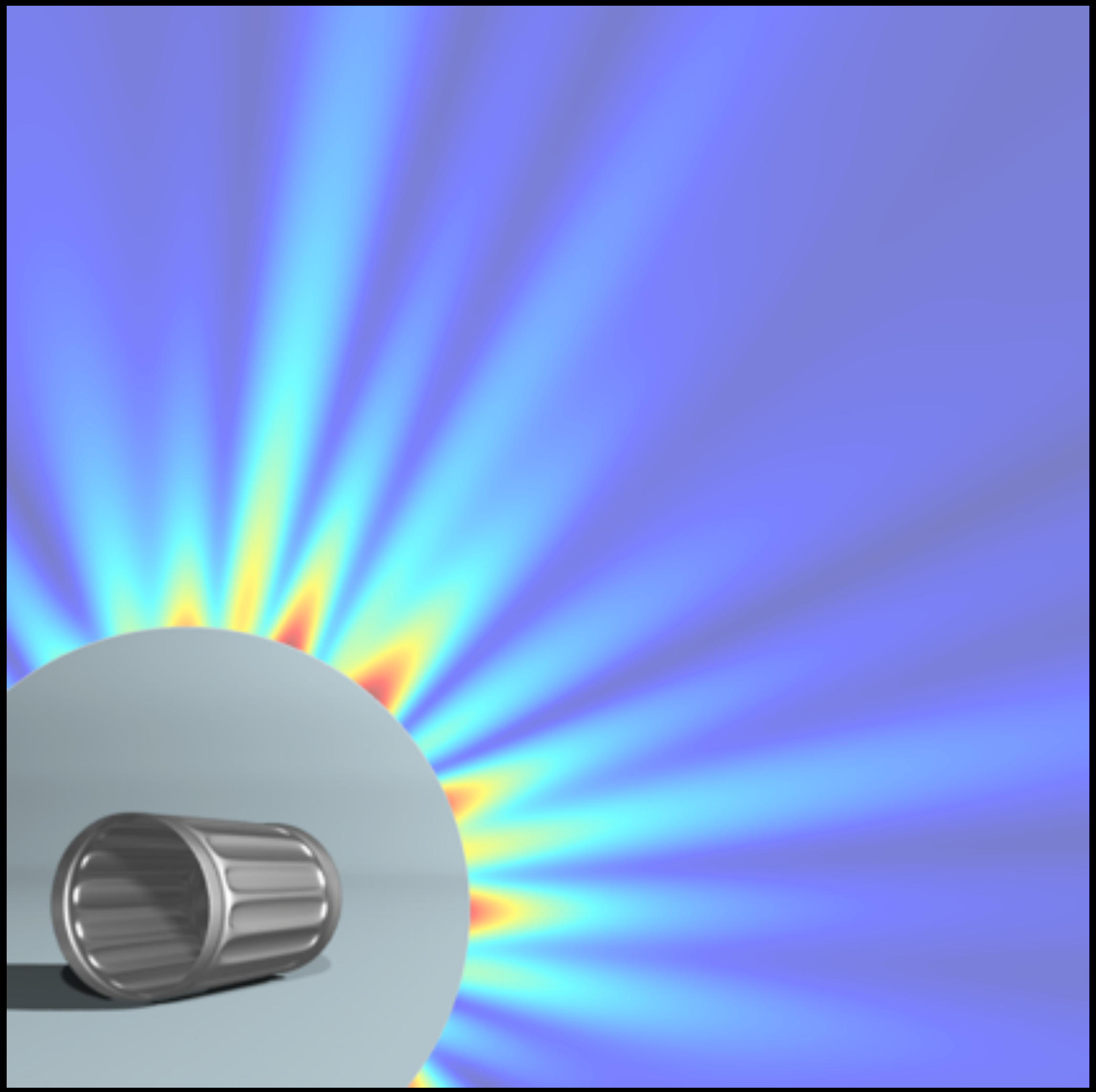

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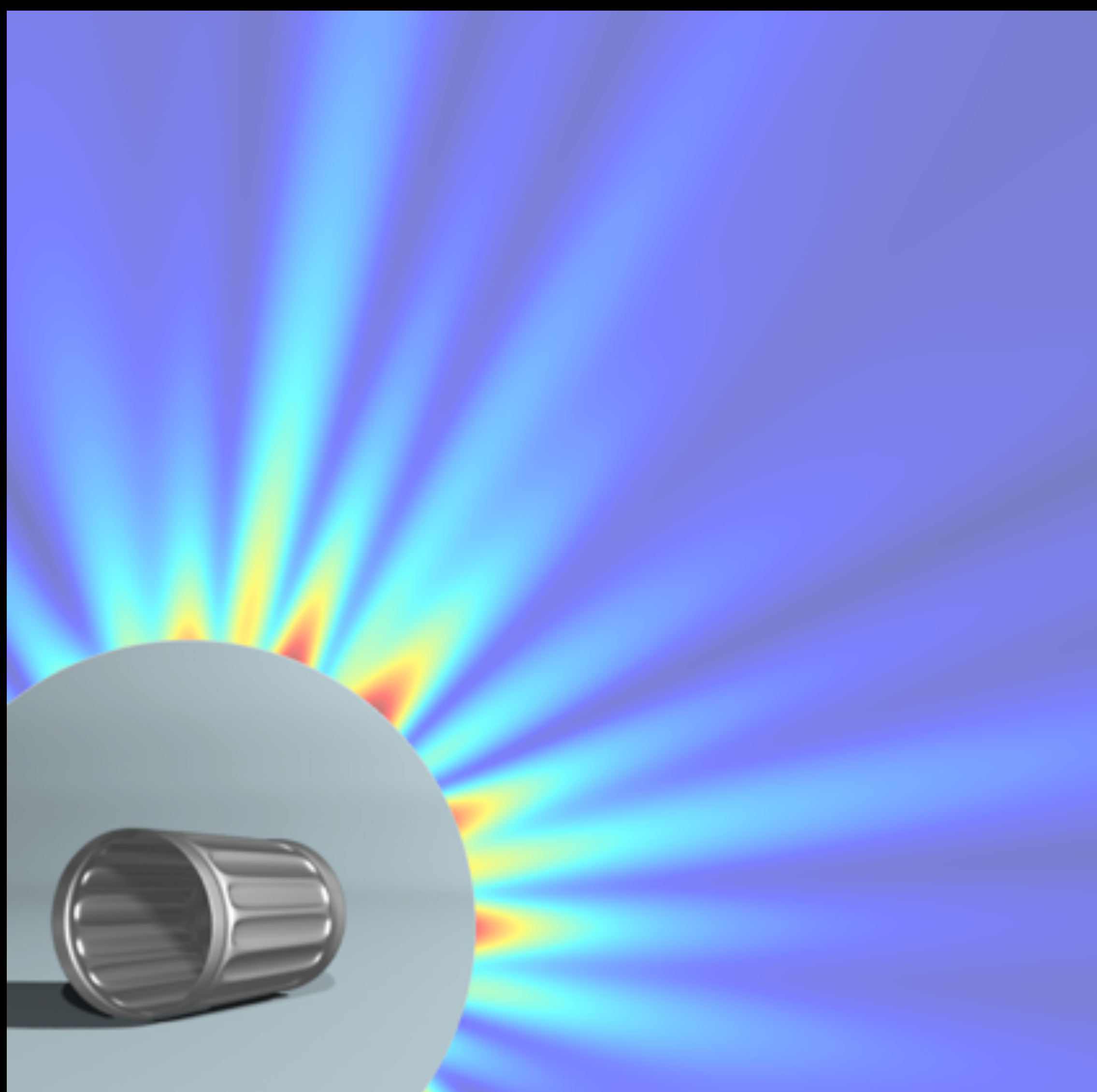

Precompute pressure samples on concentric spherical shells using fast multipole BEM
[Greengard and Rokhlin 1987; Gumerov and Duraiswami 2005]
(FastBEM implementation [Liu 2009])

Approximating Acoustic Transfer

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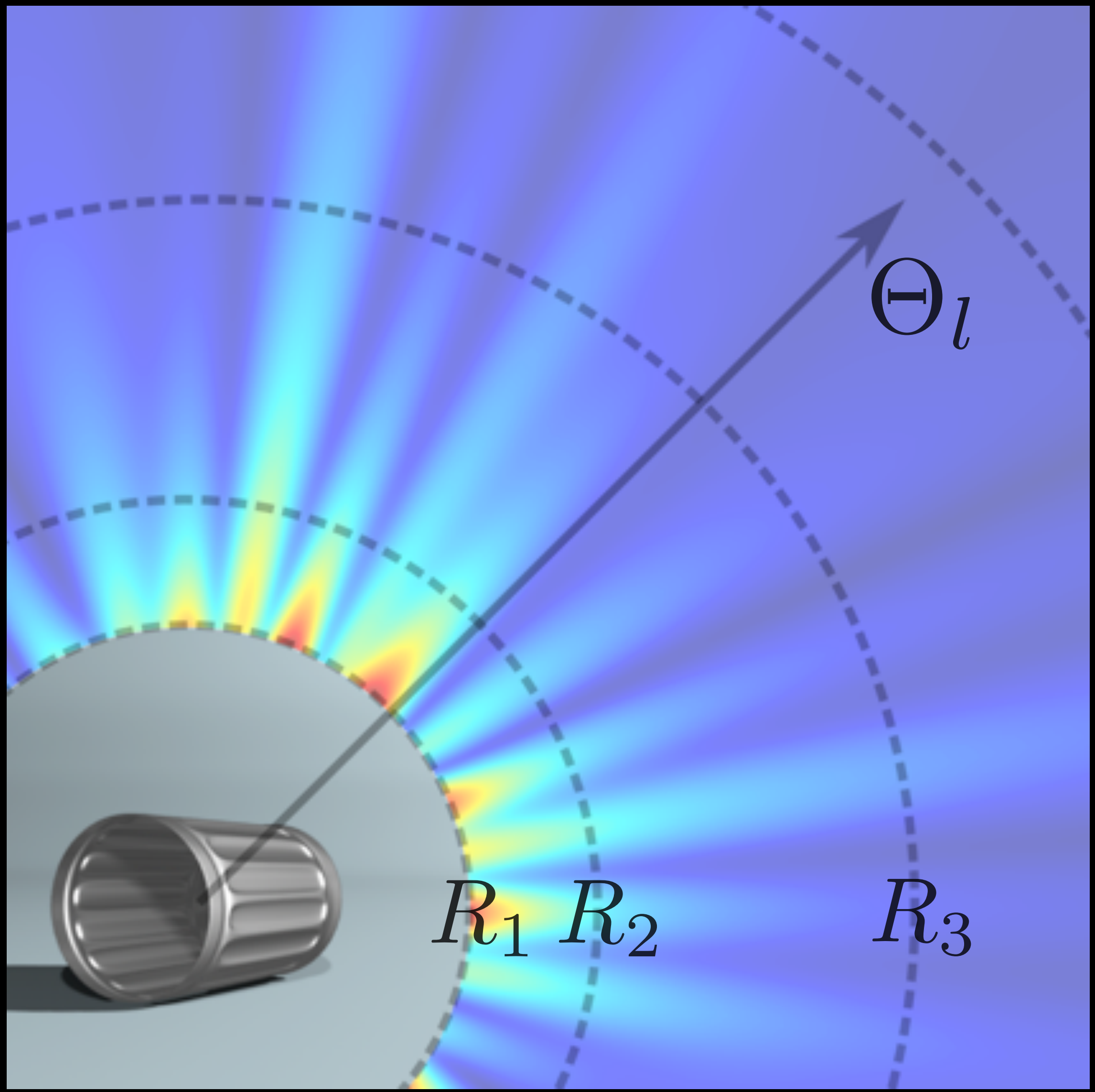

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[Greengard and Rokhlin 1987; Gumerov and Duraiswami 2005]
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Approximating Acoustic Transfer

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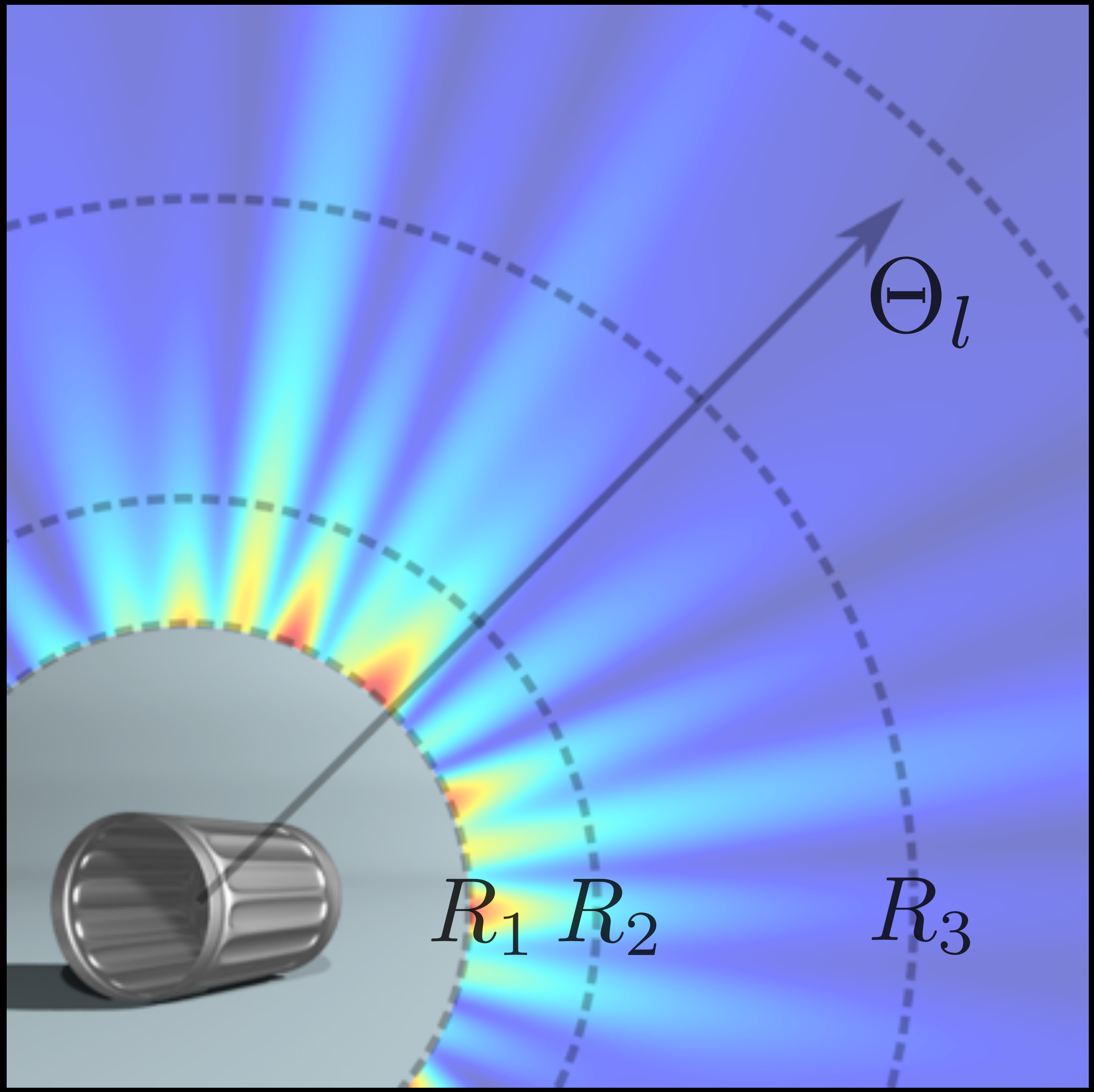

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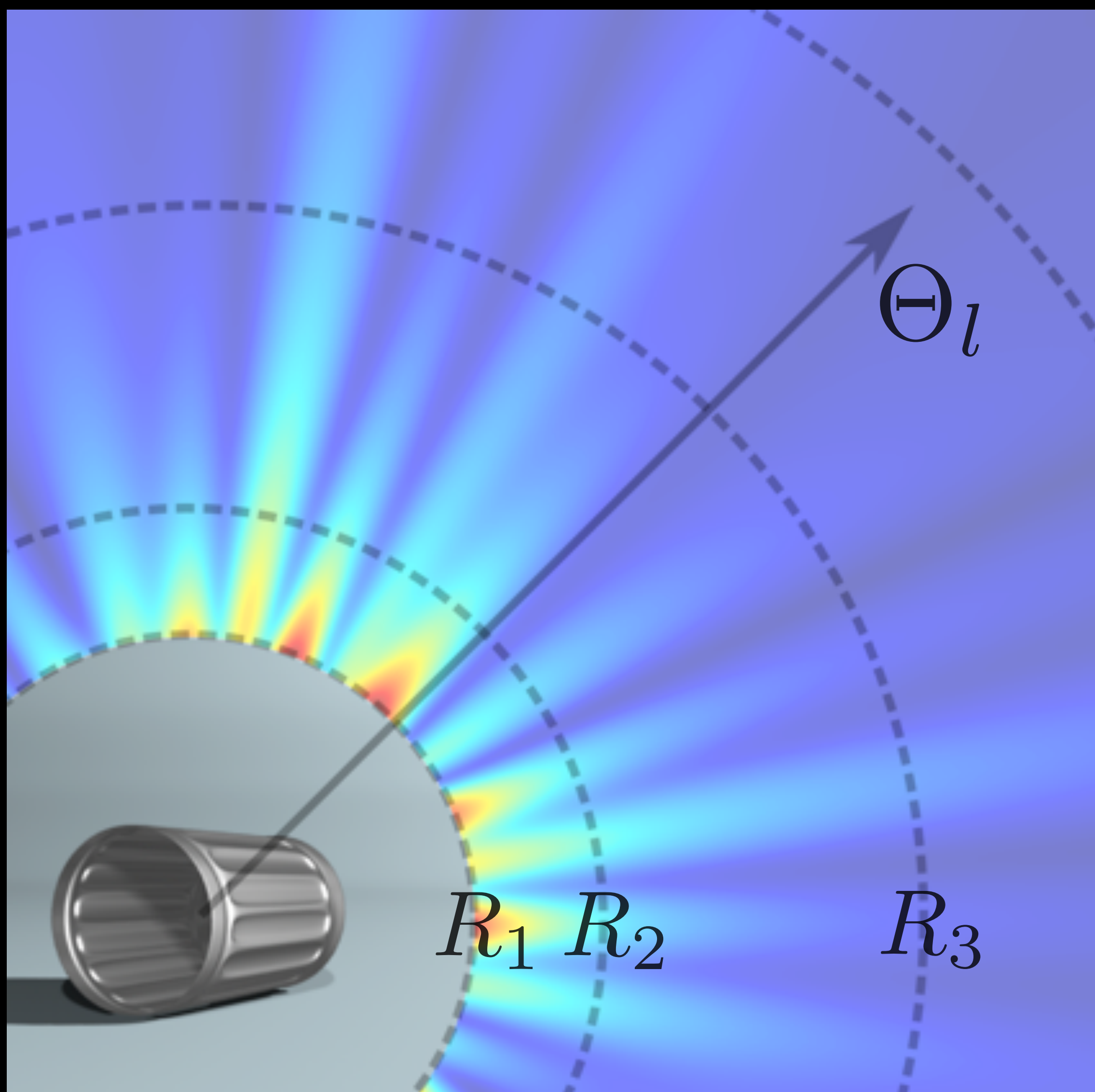

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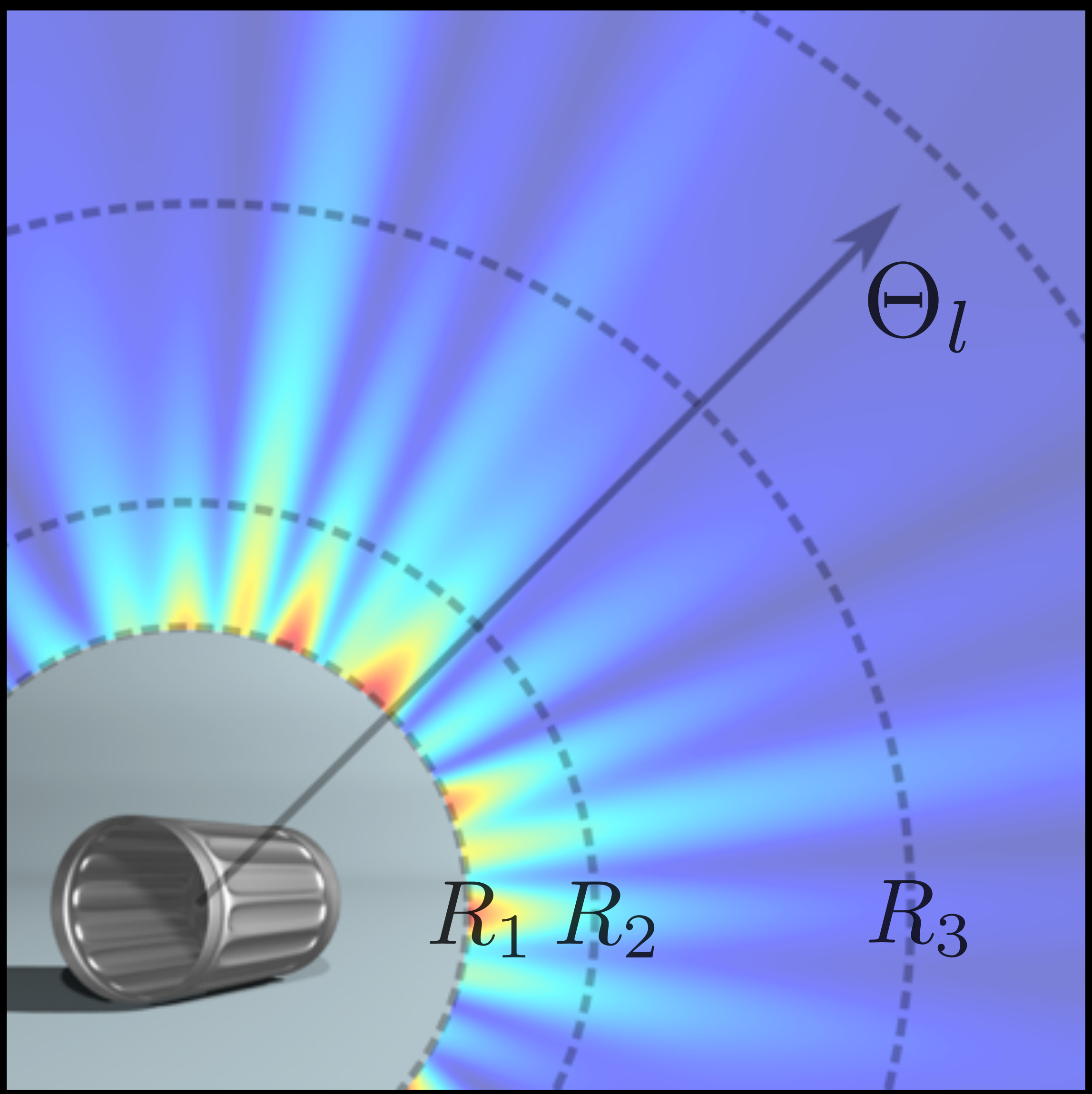

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Unknowns

Precomputed pressures

Approximating Acoustic Transfer

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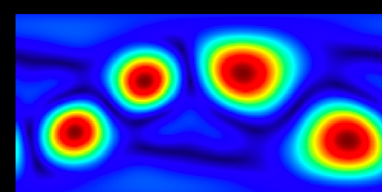
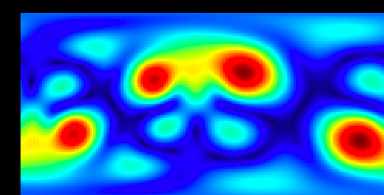
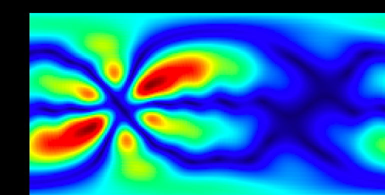
↑

$$-\frac{ie^{-ikR}}{kR}$$

Approximating Acoustic Transfer

$p(x) \sim$  $\Bigg\{$  $+$  $/\, kR$ $+ \cdots +$  $/\, (kR)^{M-1}$ $\Bigg\}$

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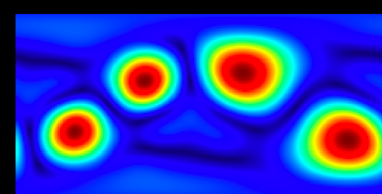
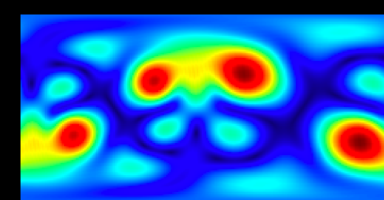
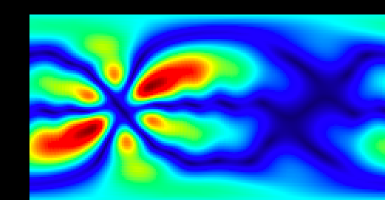
$p(x) \sim$  $\Big\{$  $+$  $/ \, kR + \cdots +$  $/ \, (kR)^{M-1} \Big\}$

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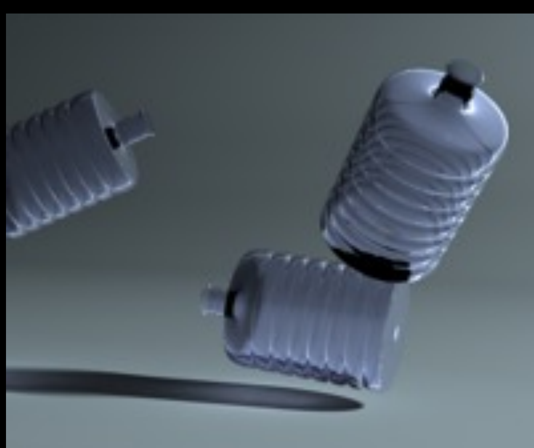
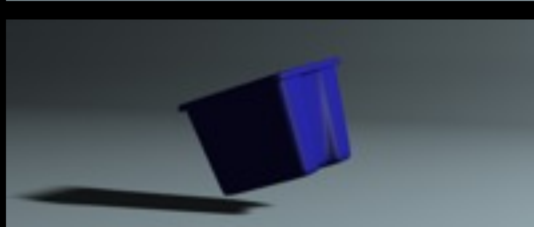
Far Field Acoustic Transfer (FFAT) Maps

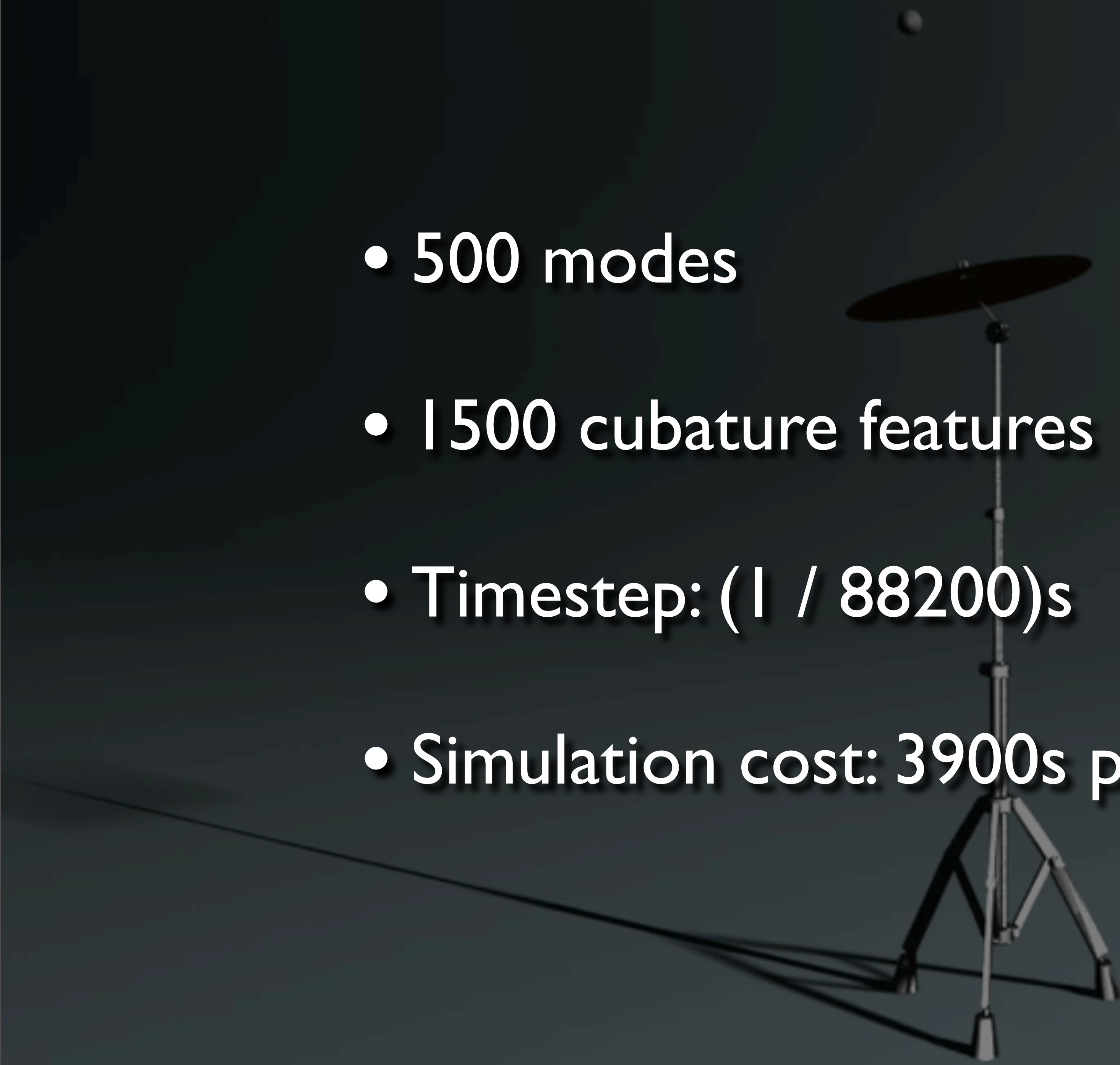
- Low-error transfer, e.g., M=4
- O(1) transfer evaluation cost

Results

	Model	Dimensions	# of triangles	# of modes	Freq. range
	Trash can	0.75m tall	78k triangles	200 modes	0.071-4.43 kHz
	Cymbal	0.50m diameter	62k triangles	500 modes	0.061-9.94 kHz
	Water bottle	0.46m tall	29k triangles	300 modes	0.116-3.59 kHz
	Recycling bin	0.61m wide	110k triangles	300 modes	0.062-2.21 kHz
	Trash can lid	0.55m diameter	34k triangles	200 modes	0.112-6.79 kHz

Results

Results



- 500 modes
- 1500 cubature features (10.7% error)
- Timestep: (1 / 88200)s
- Simulation cost: 3900s per second of audio

Results

Results

- 300 modes
- 1200 cubature features (15.7% error)
- Timestep: (1 / 44100)s
- Simulation cost: 1224s per second of audio

Results

Results

- 200 modes
- 800 cubature features (11.5% error)
- Timestep: (1 / 44100)s
- Simulation cost: 624s per second of audio

Results

Results

- 200 modes
- 800 cubature features (10.3% error)
- Timestep: (1 / 44100)s
- Simulation cost: 714s per second of audio

Results

Results

- 300 modes
- 900 cubature features (10.7% error)
- Timestep: (1 / 44100)s
- Simulation cost: 1026s per second of audio

Comparisons

Comparisons: Linear vs. Nonlinear

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1. Nonlinear dynamics + Transfer
("Harmonic Shells") (~1.5-3h per 10s of audio)

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More Results

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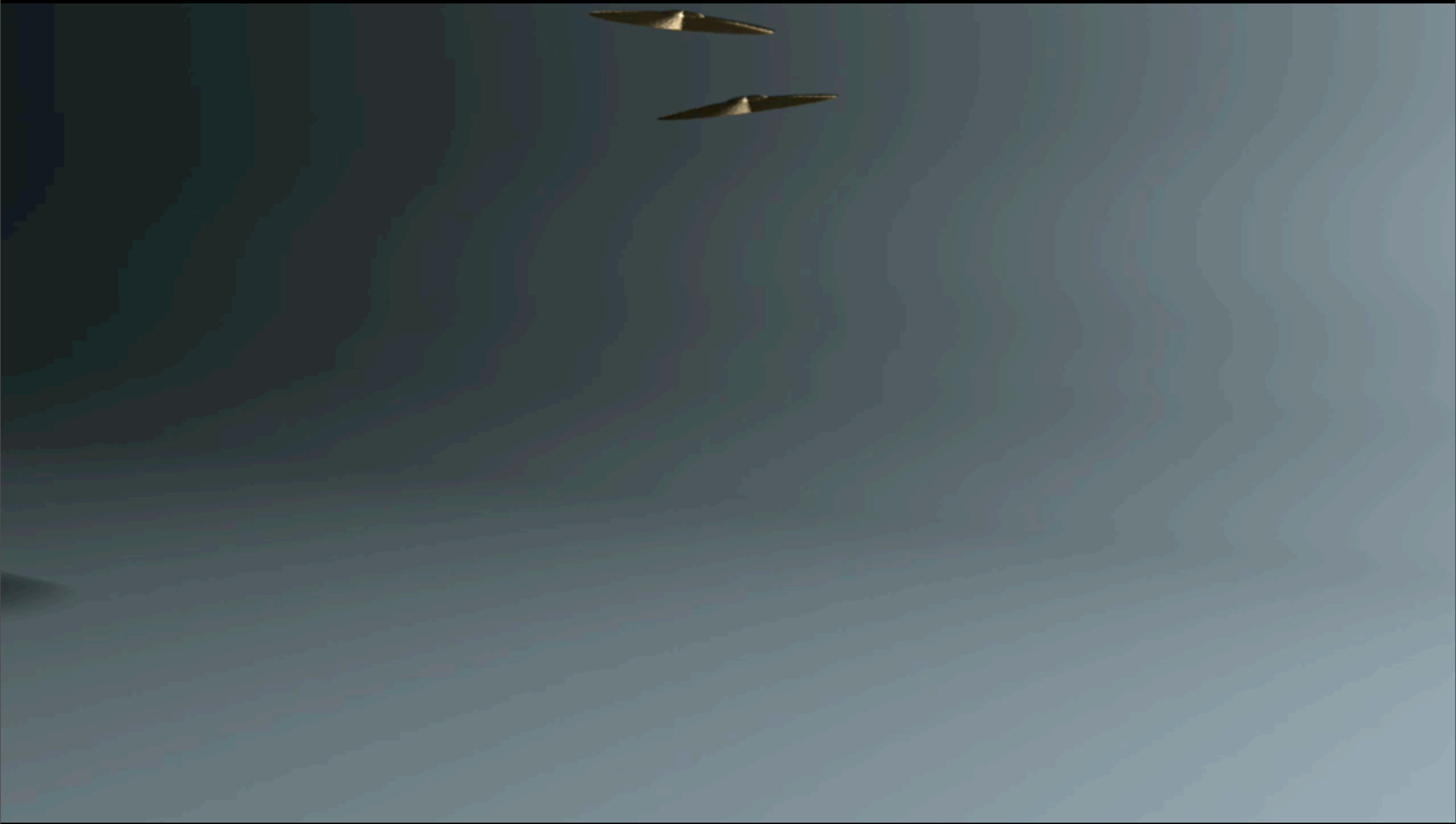

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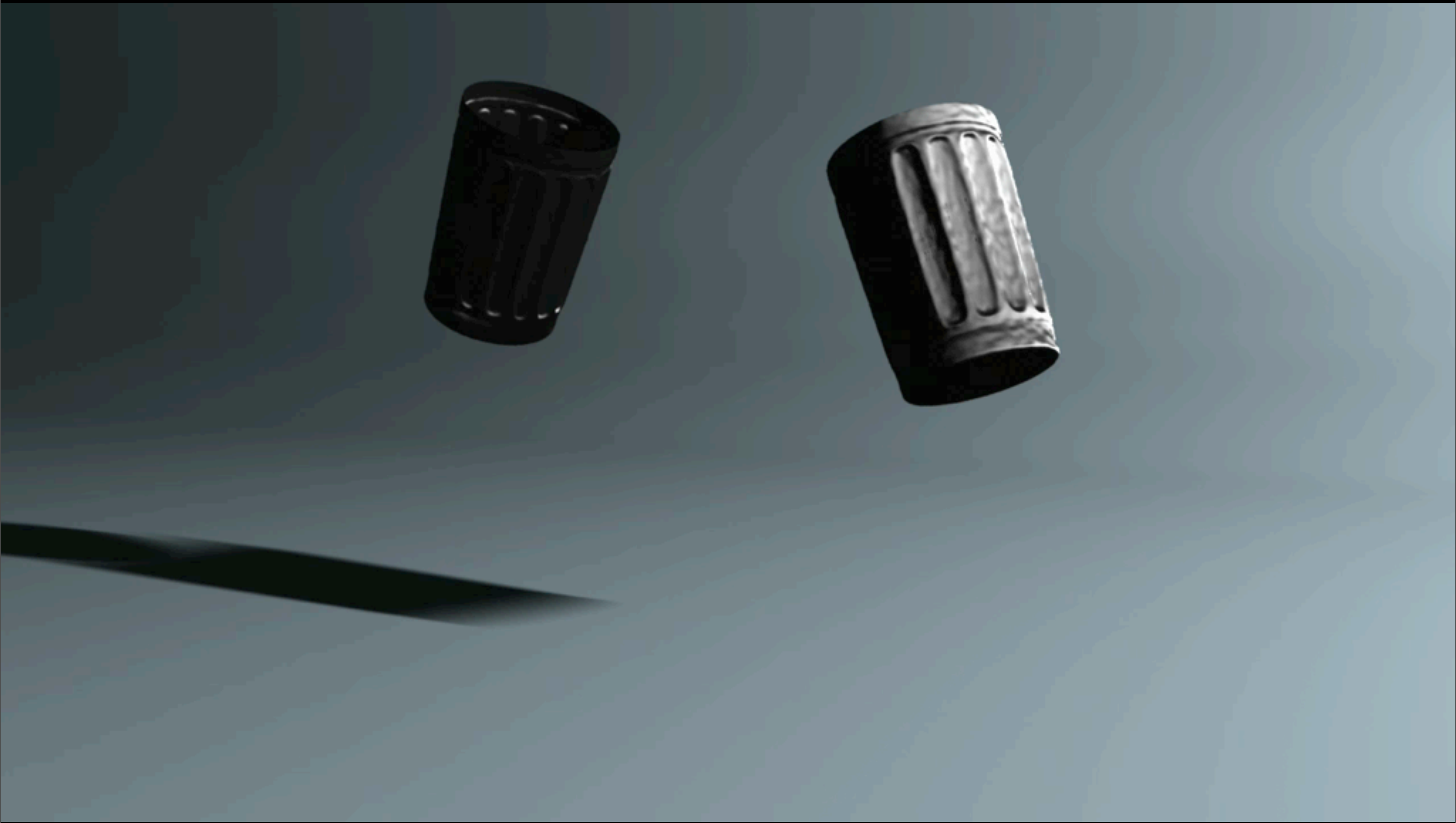

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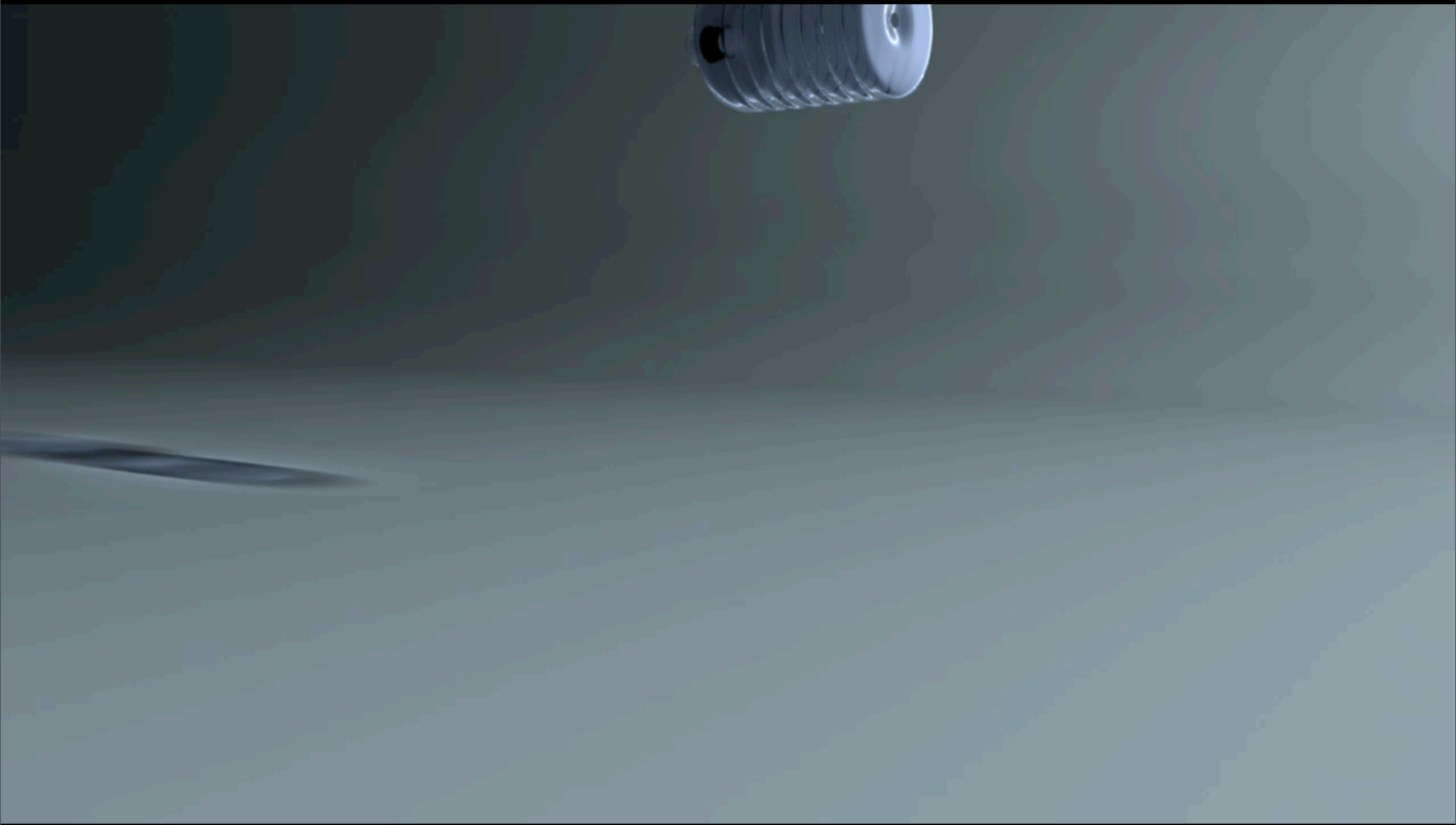

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  - Better sampling of angular space (not all directions as complex)

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    - Radiation model which takes into account mode coupling, etc.

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  - $O(r)$ for all r modes

Acknowledgements

- Anonymous Reviewers
- The National Science Foundation:
  - CAREER-0430528, EMT-CompBio-0621999, HCC-0905506
- NIH (NIBIB/NIH R01EB006615)
- Alfred P. Sloan Foundation
- Pixar
- Intel
- Advanced CAE Research, LLC (FastBEM Acoustics)
- Autodesk
- NVIDIA